

Lexical Pragmatics and the Geometry of Opposition: The mystery of **nall* and **nand* revisited

Laurence R. Horn
Yale University

The back cover blurb for Dany Jaspers' (2005) monumental recent work on the linguistic asymmetry of the Square of Opposition begins as follows:

Operators in the Lexicon begins with an old chestnut: why are there no natural single word lexicalizations for negations of the propositional operator *and* and the predicate calculus operator *all*: neither **nand* nor **nall*?

I assume that the “old chestnut” in question concerns not senses 1-5 of the *American Heritage Dictionary* entry for “chestnut” (relating to trees of the genus *Castanea*, their wood or nuts, the reddish-brown color of those trees or the horse of the same color, or a callus on a horse's foreleg) but rather sense 6: “An old, frequently repeated joke, story, or song.” In the absence of a live audience or a singing voice, I will further narrow down the intended reference to the third disjunct within sense 6, and in particular to the frequently recited story I shall describe (with apologies to the soi-disant Réage 1954) as *l'Histoire d'*O*:

In a wide variety of languages, values mapping onto the southeast, **O** vertex of the Square of Opposition are systematically restricted in their potential for lexicalization or direct expression; this asymmetry is attributable to the scalar implicature relation obtaining between the two subcontraries.

(Horn 1972: Chap. 4; Horn 1989: §4.5; Horn 2006: §5; cf. Levinson 2000: 69-71.)

The relation between the subcontraries *some* and *not all* (*some not*) has had a tortuous history. On the minority view, *some* is two-sided, with upper as well as lower bound, and is thus incompatible with *all*. Some have read this position into Aristotle—

On the Aristotelian theory...wherever the affirmative “some are” applies, the negative proposition “some are not” holds also. (Dewey 1938: 182)

Aristotle seems to think that the main function of a particular statement is to describe a situation where the corresponding universal statement is false. His reasoning seems to be: If the universal is true, why assert the particular? (Rose 1968: 41)

But this reading is not tenable, given both Aristotle's observation on the nature of the relation between the two subcontraries themselves (“Verbally four kinds of opposition are possible...but really there are only three: for the particular affirmative is only verbally opposed to the particular negative”—*Prior Analytics I*, 63b21ff.) and his explicit endorsement of the one-way subaltern entailment from **A** to **I** and from **E** to **O**:

For having shown that it belongs to all, we will have shown also that it belongs to some; similarly, if we should show that it belongs to none, we will have shown also that it does not belong to all. (Aristotle, *Topics* 109a3)

Most commentators concur; in the words of the great Arab philosopher Avicenna (ibn-Sīnā):

If you say “some men are so-and-so”, it is not necessary that some others are not so-and-so. If the proposition is about all, it is also about some.

(Avicenna/Zabeeh 1971: 24)

But even if Aristotle did not support a two-sided *some*, others did. Priority may belong to the 5th-6th century Buddhist logician Dignāga and his colleagues who, in their *hetu-cakra* or Wheel of Reasons,

do not admit four kinds of proposition like Aristotle and the Scholastics, but only three, since they interpret ‘Some S is P’ not as ‘at least some’ but as ‘at least some and not all’...This would give a logical triangle in place of the western logical square. (Bochenski 1961: §53E; cf. Tucci 1928)

Such triangles of oppositions did not surface in the West until the mid-nineteenth century, when Sir William Hamilton of Edinburgh inaugurated a debate over the proper treatment of the subcontraries. Distinguishing two senses of *some*, the INDEFINITE (*at least some*) and the SEMI-DEFINITE (*some but not all*), Hamilton (1860: 254) regarded the latter as basic: ‘Some, if not otherwise qualified, means *some only*—this by presumption.’ On this reading of the particular, the two statements *Some men are learned* and *Some men are not learned* are not only (as for Aristotle) compatible, given that their conjunction is consistent, but logically indistinct. The purported opposition between the two subcontraries, charged Hamilton (1860: 261), was ‘only laid down from a love of symmetry, in order to make out the opposition of all the corners in the square of opposition...In reality and in thought, every quantity is necessarily either all, or none, or some. Of these the third...is formally exclusive of the other two.’

As was his wont, Augustus de Morgan was unimpressed with his rival’s stance, although he did concede the existence of what we would now view as pragmatic upper-bounding. Some sample passages (emphasis added):

In common conversation the affirmation of a part is meant to imply the denial of the remainder. Thus, by ‘some of the apples are ripe’, it is always [sic] intended to signify that some are not ripe. (De Morgan 1847: 4)

Some, in logic, means *one or more, it may be all*. He who says that *some are*, is not to be held to mean the rest are not. ‘Some men breathe’...would be held false in **common language** [which] usually adopts the complex particular proposition and implies that some are not in saying that some are.

(De Morgan 1847: 56)

With logicians the word *some* has in all time been no more than a synonym of *not–none*: it has stood for *one or more, possibly all*. **With the world at large** it is sometimes *possibly all*, sometimes *certainly not all*, **according to the matter spoken of**. But with the logician “some are” is merely and no more than the contradictory of “none are”...[S]ome equally contains some–certainly–not–all and some–possibly–all. (De Morgan 1861: 51)

As recognized by both De Morgan and Mill (another captain of the anti-Hamilton forces), the possibility of upper bounding is subject not only to the vagaries of context and speech level but more specifically to the epistemic insecurity of the speaker:

There are three ways in which one extent may be related to another...: complete inclusion, partial inclusion with partial exclusion, and complete

exclusion. This trichotomy would have ruled the forms of logic, **if human knowledge had been more definite**...As it is, we know well the grounds on which predication is not a trichotomy, but two separate dichotomies.
(De Morgan 1858: 121)

No shadow of justification is shown...for adopting into logic **a mere sous-entendu of common conversation** in its most unprecise form. If I say to any one, "I saw some of your children today", he might be justified in inferring that I did not see them all, **not because the words mean it**, but because, if I had seen them all, it is most likely that I should have said so: **even though this cannot be presumed unless it is presupposed that I must have known whether the children I saw were all or not.**
(Mill 1867: 501)

The result is essentially the two-stage quantity implicature procedure reconstructed within neo-Gricean pragmatics:

The use of a weaker value (e.g. *some*, *most*) suggests that **for all the speaker knows** a stronger value—and especially the strongest value—on the same scale (*all*) could not have been substituted *salva veritate*. Speaker *a*'s utterance of *...W...* implicates not $K_a\text{-}(S)$, i.e. that *a* knows that the stronger counterpart *...S...* is false, but only (*ceteris paribus*) that $\neg K_a(S)$. [cf. e.g. Horn 1972, 1989, 2006]
☞ *a* says "*...W...*" \rightarrow_1 for all *a* knows $\neg\text{...S...}$ \rightarrow_2 (*ceteris paribus*) *a* knows that $\neg\text{...S...}$

Mill's allusion to a tacit principle that mandates the speaker to use the stronger *all* in place of the weaker *some* when possible, while inviting the hearer to draw the corresponding inference when the stronger term is not used, is echoed even by one of Hamilton's sometime supporters:

Whenever we think of the class as a whole, we should employ the term All; and therefore when we employ the term Some, **it is implied** that we are not thinking of the whole, but of a part as distinguished from the whole—that is, of a part only.
(Monck 1881: 156)

The idea that *some* should be assigned a two-sided meaning rather than, or along with, the traditional one-sided interpretation did not die with Hamilton. Ginzberg (1913, 1914) carried the quarrel across the Channel, jettisoning the square of opposition for a triangle of contraries with vertices representing *all*, *none*, and *exactly some*—'quelques et rien que quelques'. But Couturat (1913, 1914), only too happy to play De Morgan to Ginzberg's Hamilton, sought to dissuade his countryman from following 'le plus mauvais des logiciens' in collapsing the two distinct subcontraries into one basic proposition which is in fact a logical conjunction; he argues that the classical system cannot be perfected by adopting 'précisions' that are inconsistent with its very spirit.

John Neville Keynes echoed Mill in observing (1906: 202-3) that a speaker whose knowledge is incomplete cannot use *some S's are P* with the meaning 'some only'. Unfortunately, many logicians "have not recognized the pitfalls surrounding the use of *some*. Many passages might be quoted in which they distinctly adopt the meaning—*some, but not all*."

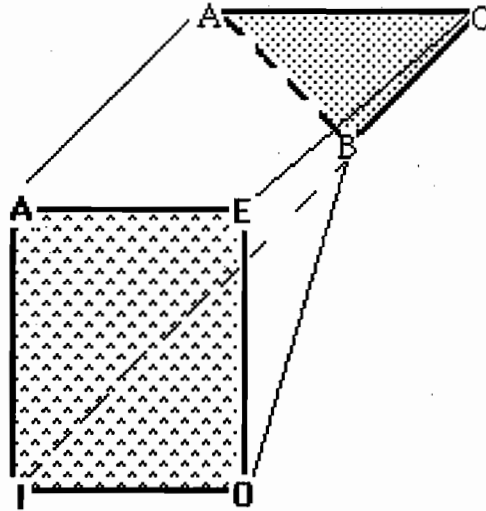
To which the great Danish linguist Jespersen (1924: 324) retorted "in the name of common sense", why do logicians "dig such pitfalls for their fellow-logicians to fall in"? Jespersen consequently proselytizes for the "tripartition" of operators in (1) (cf. De

Morgan's functional "trichotomy" above) and implicitly proposes his own Triangle of Opposition (1917: Chapter 8) as indicated in (2)

- (1) A: all everybody always everywhere necessity must/need command
 B: some/a somebody sometimes somewhere possibility can/may permission
 C: none/no nobody never nowhere impossibility cannot prohibition

The effect, as with Ginzberg (or Dignāga), is to triangulate the Square:

(2)



But while Jespersen's B category, the nadir of his Triangle, represents a semantic conjunction (or neutralization) of the I and O vertices of the traditional Square, it has the lexical membership of the I vertex (*some, possible*). On logical, epistemological, and discourse grounds the identification of I and O is ultimately untenable, precisely for the traditional reason that the former provides the contradictory of E, the latter of A. (See Horn 1973 for more on these debates in the light of neo-Gricean pragmatics.)

With characteristic insight, Sapir (1930: 21) opted for a solution midway between the classical Square and the Jespersenian Triangle. His particular subcontraries are neither semantically bilateral nor strictly unilateral. (As usual, emphasis is added.)

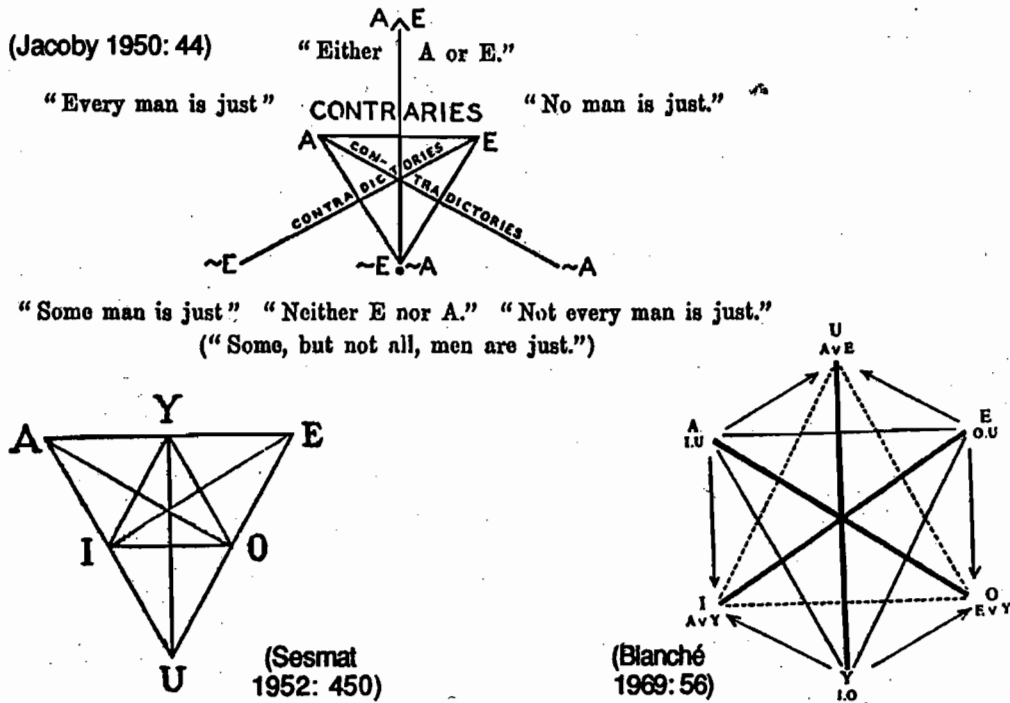
'Not everybody came' **does not mean** 'some came', **which is implied**, but 'some did not come'. **Logically**, the negated totalizer [*not every*] should include the totalized negative, i.e. opposite or contrary [*none*], as a possibility, but **ordinarily** this interpretation is excluded.

Note especially Sapir's use of *is implied* (vs. *means*) and his qualifier *ordinarily*, emphasizing the essential role of the context in licensing the implication in question.

An expanded development of the triangle was launched in the early 1950's, appropriately enough by three philosophers working independently but exploiting essentially the same geometry. For Jacoby (1950), Sesmat (1952), and especially Blanché (1952, 1953, 1969), the square and triangle in (4) can be combined to form a Hexagon of Opposition on which the diametrically opposed terms are contradictories:¹

¹ It will be noticed that Sesmat's hexagon has the Y above and the U below, as does the somewhat sketchier model of Hegenberg (1957). I opt here for Blanché's vowel system for its mnemonic value, the I O U serving to acknowledge the incurring of an informational debt. Von

(3)

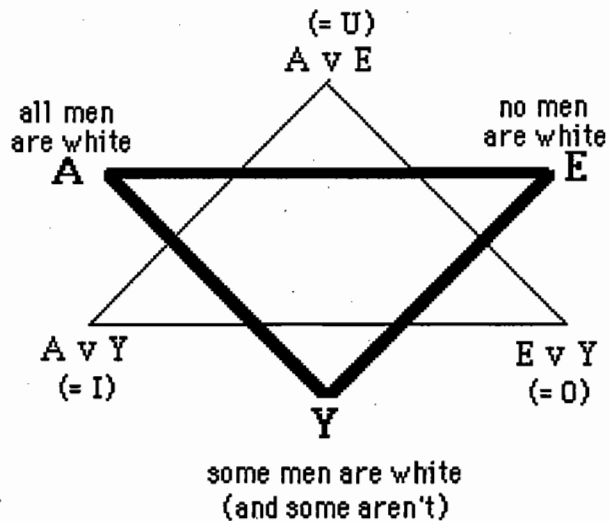
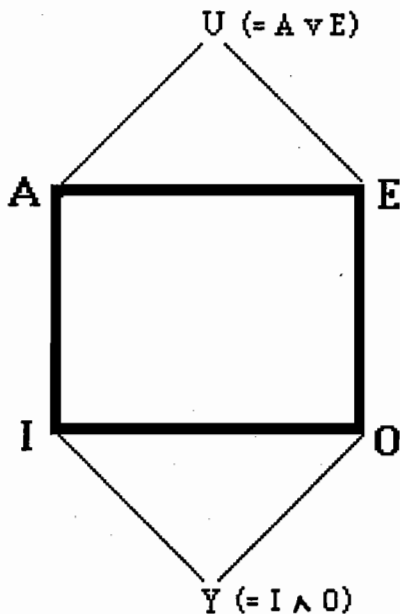


In earlier work (Horn 1990), I suggested reconfiguring the hexagons in (3) as a Logical Star of David, in which the triple of pairwise contraries $\langle A E Y \rangle$ (exactly one of which must be true in any context) is superimposed on the triple of pairwise subcontraries $\langle I O U \rangle$ (exactly one of which must always be false).

(4) Logical hexagon



(5) Logical Magen David



Wright (1951) presupposes a logical pentagon, with a nadir (= our Y) for the conjunction of I and O but no apex (= U) for its contradictory, as in (8) below.

The (neo-)Gricean approach derives the relationship between the subcontraries by means of a pragmatic principle variously defined in terms of strength or quantity:

One should not make the (logically) lesser, when one could truthfully (and with greater or equal clarity) make the greater claim.

Strawson (1952: 178-9), with acknowledgments to 'Mr H. P. Grice'

One should not make a weaker statement rather than a stronger one unless there is a good reason for so doing.

Grice's own 'first shot' (1961: 132)

Make your contribution as informative as is required (for the current purposes of the talk-exchange).

Grice's first MAXIM OF QUANTITY ([1967] 1989: 26)

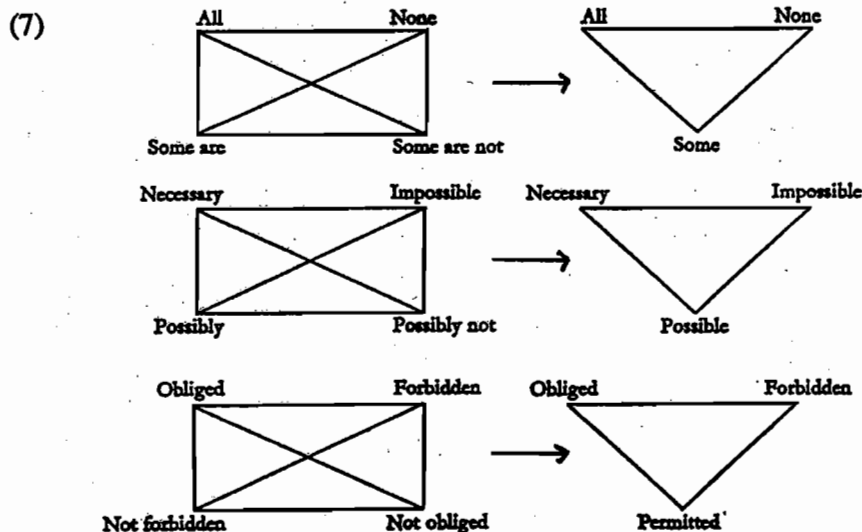
Make the strongest possible claim that you can legitimately defend!

Fogelin's RULE OF STRENGTH (1967: 20)

This was clearly an idea whose time had come. Fogelin (1967: 20-22) applies his rule of strength, devised contemporaneously with Grice's William James lectures, to the formulation of rules of use for the subcontraries:

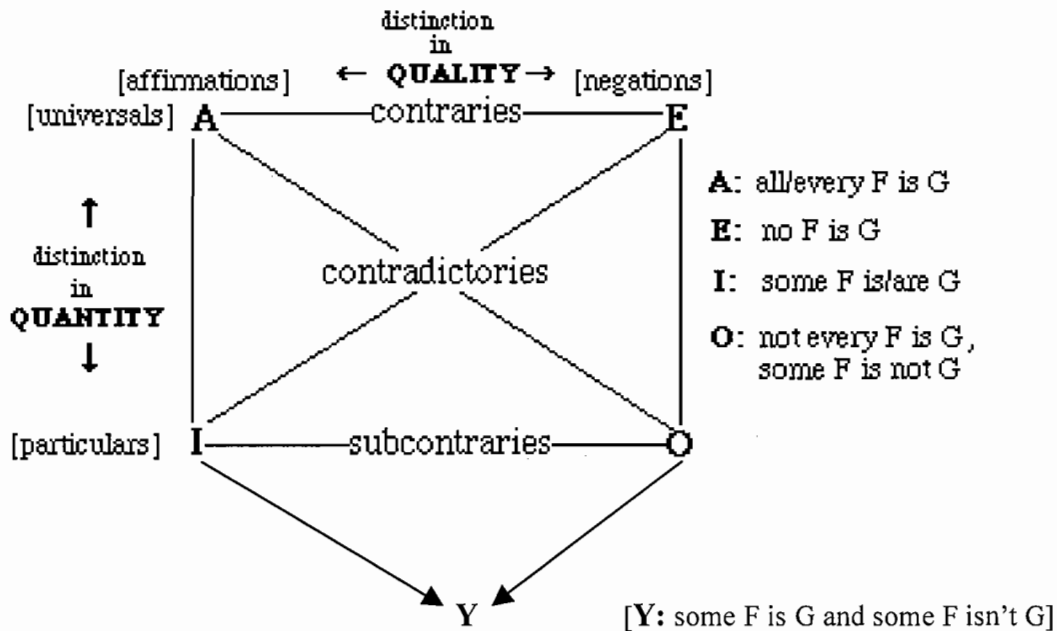
- (6) (i) Do not employ an **I** or an **O** proposition in a context where you can legitimately employ an **A** or an **E** proposition...The *use* of one subcontrary typically suggests the appropriateness of using the other.
- (ii) Do not affirm one subcontrary if you are willing to deny the other.
- (iii) Subcontraries tend to collapse together.

Based on these rules, Fogelin takes his own turn at beating Squares into Triangles:



But Fogelin's triangles—unlike those of the Dignāga-Jespersen-Jacoby-Sesmat-Blanché tradition(s)—are pragmatically derived and not semantically driven. The move from **I** and **O** to their conjunction **Y**, as in the pentagonalized Square in (8), is context dependent; it is not that the subcontraries are equipollent but that they will tend (*ceteris paribus*) to result in a speaker's communication of the same state of affairs.

(8) EXTENDED SQUARE OF OPPOSITION



In fact, it can be argued that what we need is not so much a triangle or a pentagon as a defective three-cornered square², given that in a wide variety of languages those values mapping onto the southeast corner of the Square are systematically restricted in their potential for lexicalization. This crucial asymmetry was perhaps first recognized by St. Thomas Aquinas, who observed that whereas in the case of the universal negative (**A**) “the word ‘no’ [*nullus*] has been devised [*sic!*] to signify that the predicate is removed from the universal subject according to the whole of what is contained under it”, when it comes to the particular negative (**O**), we find that

there is no designated word, but ‘not all’ [*non omnis*] can be used. Just as ‘no’ removes universally, for it signifies the same thing as if we were to say ‘not any’ [i.e. ‘not some’], so also ‘not all’ removes particularly inasmuch as it excludes universal affirmation.

(Aquinas, in *Arist. de Int.*, Lesson X, Oesterle 1962: 82-3)

Thus alongside the quantificational determiners *all*, *some*, *no*, we never find an **O** determiner **nall*; corresponding to the quantificational adverbs *always*, *sometimes*, *never*, we have no **nalways* (= ‘not always’, ‘sometimes not’). We may have equivalents for *both* (*of them*), *one* (*of them*), *neither* (*of them*), but never for **noth* (*of them*) (= ‘not both’, ‘at least one...not’); we find connectives corresponding to *and*, *or*, and sometimes *nor* (= ‘and not’), but never to **nand* (= ‘or not’, ‘not...and’). Schematically:

² I pass over another issue arising in the mapping of quantificational values onto the Square: the role of existential import. Which (if any) of the four statement forms entail or presuppose that the set over which the quantifier ranges has members and how does this affect the subaltern and other relations? In particular, if (as in the modern tradition) *all* is import-free while *some* is not, does this vitiate the Square, as Fogelin (1978) maintains? The fact that other operators (binary connectives, adverbs, modals, deontics) can be mapped onto the Square in the usual way where no considerations of existential import arise make this move as unappealing as it is unnecessary. This leaves a number of options open for dealing with questions of import and quantification and their relation to the Square; see Horn (1997, 2007) for elaboration.

(9) *L'Histoire d'*O*

	DETERMINERS/ QUANTIFIERS	QUANT. ADVERBS	BINARY QUANTIFIERS	CORRELATIVE CONJUNCTIONS	BINARY CONNECTIVES
A:	all α , everyone	always	both (of them)	both...and	and
I:	some α , someone	sometimes	one (of them)	either...or	or
E:	no α , no one (=all-/-some)	never (=always-)	neither (of them) (=both-/-either)	neither...nor (=[both...and]-)	nor (=and-)
O:	*nall α , *neverone (=some-/-all)	*nalways (=-always)	*noth (of them) (=either-/-both)	*noth...nand (=[either...or]-)	*nand (=and-/-or)

As argued more fully elsewhere (Horn 1972; Horn 1989: §4.5), the motivation for this asymmetry is pragmatic: the relation of mutual non-logical inference between the positive and negative subcontraries results in the superfluity of one of these subcontraries for lexical realization; given the functional markedness of negation (see Horn 1989 for a comprehensive review), the superfluous, unlexicalized subcontrary will always be **O**.

We shall see below that the missing **O** phenomenon extends to the modals and deontics, and is also reflected in the general tendency toward **O**→**E** drift, wherein lexical items or collocations associated by their compositional form or etymology with the **O** corner of the square move inevitably northward toward **E**. But we will first elaborate the nature of the asymmetry among the quantifiers and related adverbs and connectives in (8), where its operation is particularly robust.

This effect is especially striking when it overcomes apparent morphology. The evidence is even clearer in Old English, where the lexical item *nalles*, *nealles*—while appearing to challenge the constraint blocking **nall*-type determiners—is in fact attested only with the value ‘no, not, not at all’, never ‘not all’. Other OE quantificational expressions include *næfre* ‘never’, *næðor* ‘neither, nor’, *náht* ‘nothing’, *nán* ‘no one, none’, and *náhwær* ‘nowhere’, all occupying the **E** rather than **O** slot. Along the same lines, Jaspers (2005: 150) points to the Dutch adverb *nimmer*—lit., ‘nalways’, but actually denoting ‘never.’

Consider the binary connectives, which have been situated on the Square since the medievals—cf. Ashworth (1974: 148, 167). Eloquent testimony to the persistent cross-linguistic exclusion of **nand* is provided by the Latin *neque*, which the *nealles* effect: its **O** morphology (‘not’ + ‘and’) belies its **E** (‘and not’) semantics, i.e. ‘neither, nor’ (Jaspers 2005: 150). While the *nand gate*, widespread in electronic circuitry does indeed cover precisely the taboo **O** vertex, this merely underlines the absence of any such lexical item from natural languages. (But see Seuren 2006: fn. 8 for a potential counterexample.)

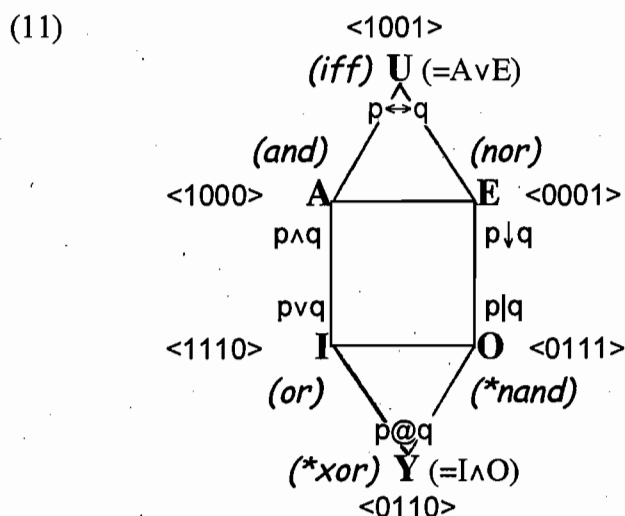
The modern English connective *nor* is essentially an **E** value, but it is usually restricted to negative polarity contexts—although an untriggered *nor* is not quite as restricted or marginalized as sometimes claimed, as in this passage:

The two values standardly lexicalized occupy the **A** and **I** slots, represented in English by *and* and *or* respectively. If a third position is represented, typically through negative incorporation, it maps onto the **E** vertex. Jespersen (1917: 108) cites Old English (and Old Norse) *ne*: as a negative phrasal conjunction ‘looking before and after’, so that *suð ne norð* in *Beowulf* stands for ‘neither south nor north’. German offers a (rarely invoked) modern equivalent: *in Wasser noch in Luft* ‘neither in the water nor in the air’. (Horn 1989: 256)

But in fact similar occurrences of a Janus-faced *nor* are attested in both literary and colloquial English:³

- (10) 1872 TENNYSON Great brother, thou nor I have made the world. [OED]
 1954 W. FAULKNER A world such as caesar nor sultan nor khan ever saw [OED]
 “Kent Smith, nor anyone from that office was present”
 “I, nor my management, have ever had any kind of problem with creating a gay character...”
 “I, nor my host, nor my file server, nor my ISP are responsible for what you do with the patches and ROM images found on this site.”

Crucially, while joint denial ($p \downarrow q$, $\langle 0001 \rangle$) may be lexicalized, the Sheffer stroke ($p | q$, $\langle 0111 \rangle$) cannot be. But what of exclusive disjunction, $\langle 0110 \rangle$, occupying the Y position in the hexagon in (10)?



In fact, there true exclusive disjunction exists—but not in natural language. As defined in set-theoretic terms, $x \in A \text{ XOR } B$ iff $x \in A \cup B$ & $x \notin A \cap B$. But no bona fide representatives of the exclusive disjunction operator have surfaced in **natural** language (Horn 1972, 1989; Gazdar & Pullum 1976).

There are, to be sure, pretenders to the throne. While Collinson (1937), Quine (1940), Geach (1972), and even Blanché (1969: 145) assert or presuppose that Lat. *aut* plays exclusive (Y) to *vel*'s inclusive (I)⁴, or when similar claims are made about Finnish or Welsh, closer inspection shows it just ain't so (Horn 1989: 224-26). Latin *aut*, for example (the source of my @ 'A(U)T' notation for exclusive disjunction), was used to oppose two mutually exclusive ('p or q, it matters which'), while *vel* (the source of the inclusive v connective) was used in free choice contexts ('p or q, it doesn't matter which'). *Aut aut*, damned "ambiguity"!

³The first two examples are cited in the OED, while the last three were provided by Arnold Zwicky and Beverly Flanigan in a thread on this topic on the ADS-L listserv.

⁴Here, for example, is Quine (1940: 5) on the "ambiguity": "We must decide whether 'or' is to be construed in an *exclusive* sense, corresponding to the Latin 'aut', or in an *inclusive* sense, corresponding to the Latin 'vel'."

The pragmatic approach for deriving so-called exclusive disjunction (Horn 1972, 1989) was prefigured in Archbishop Whately’s comments on the two disjuncts in e.g. *Virtue tends to procure us either the esteem of mankind or the favor of God* and in the more general (and still apt) considerations of Mill:

[F]rom one being affirmed we are not authorized to deny the other. Of course, we are left to **conjecture** in each case, **from the context**, whether it is to be implied that the members are or are not exclusive.

(Whately 1848: 106, emphasis added)

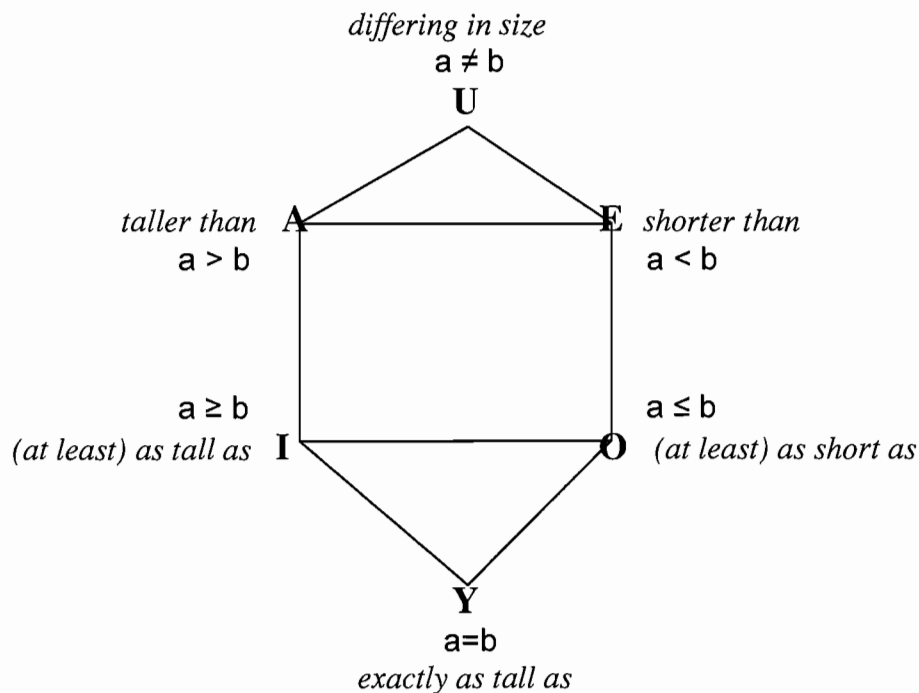
When we say A is either B or C we imply that it cannot be both...If we assert that a man who has acted in a particular way must be either a knave or a fool, we by no means assert, or intend to assert, that he cannot be both.

(Mill 1867: 512)

On the neo-Gricean account, **nand* (and **xor*) will be excluded for the same reason as **nall*: given that *p or q* tends to implicate ‘[for all S knows] not both p and q’, the closed set of connectives need admit just the one **I** vertex disjunctive connective.

The asymmetry of (9) extends from the quantificational operators and connectives to other values that can be assigned a logical geometry. Consider, for example, the arithmetical and (in)equalities and adjectival comparatives and equatives in (12).

(12)



While the **A**, **E**, and **I** values have an unrestricted distribution (Chris can be *taller than*, *shorter than*, or *as tall as* Robin regardless of their respective heights), the use of the **O** value (e.g. *Chris is as short as Robin* presupposes that Chris and Robin are (relatively) short. Earl may be as tall as Muggsy even if they’re both unusually short for their comparison set, but Shaq can’t be as short as Yao if they’re both 7-footers, given the marked nature of the “negative” adjective.

The lexicalization asymmetry of the Square extends to the modals and deontic operators (cf. Béziau 2003 for relevant discussion), as illustrated by the fact that the inflected negative in (13b) only allows wide scope (**E** vertex) negation, i.e. the Roman

Catholic reading, while its unlexicalized counterpart in (13a) allows both wide-scope (Catholic) and narrow-scope (Episcopalian) readings of the negation, the **O** version brought out by interpolation as in (14).

- (13) a. A priest can not marry. $[-\diamond(\mathbf{E}) \text{ or } \diamond\neg(\mathbf{O})]$
 b. A priest {can't/cannot marry}. $[\text{only } \neg\diamond(\mathbf{E}), \text{ not } \neg\diamond(\mathbf{O})]$

(14) A priest can {always/if he wishes/of course} not marry.

The same asymmetry holds for *could not*, although it only shows up in the inflected form, since there is no *couldnot* orthographic lexicalization alongside *cannot*:

- (15) a. You could not work hard and still pass. $[-\diamond(\mathbf{E}) \text{ or } \diamond\neg(\mathbf{O})]$
 b. You couldn't work hard and still pass. $[\text{only } \neg\diamond(\mathbf{E}), \text{ not } \neg\diamond(\mathbf{O})]$

What of **A**-vertex modals? *Must* and *should*, whether understood logically, epistemically, or deontically, can only incorporate an inner negation, resulting in an **E** (or, in the case of *should*, **E**-like) meaning:

- (16) a. You mustn't go. = 'you must [not go] $[\Box\neg(\mathbf{E}), \text{ not } \neg\Box(\mathbf{O})]$
 b. You shouldn't go. = 'you should [not go] $[\text{basically } \Box\neg(\mathbf{E})]$

To be sure, the **A** modal *need* yields a lexicalized **O** value, as in (17a); in precisely this case, an **E** value is unavailable because of the negative polarity status of modal *need* (cf. (17b,c). (Analogous distributional facts hold for Dutch *hoeven* and Ger. *brauchen*.)

- (17) a. She needn't go. = 'not [she need go]' $[\text{only } \neg\Box(\mathbf{O})]$
 b. She {needs to/*need} go.
 c. %Need you go?

It might also be noted that *needn't* is restricted by semantics (it tends to be deontic) and register (it tends to be constrained to high or written style).

In some languages, including Turkish, ASL (American Sign Language), and LSF (Langue signée française), we find an opaque **E**-valued modal negation that is synchronically distant from both possibility and necessity (cf. Shaffer 2002); its **O** counterpart (\approx *needn't*) is transparent and non-lexicalized.

As we see from (17) vs. (13)-(16), while the asymmetry in lexicalizing complexes associated with the **A**, **I**, and sometimes **E** vertices as against **O** is equally exhibited across lexical domains, some domains are more equal than others (cf. van der Auwera 2001 for related discussion). The degree of asymmetry varies according to how closed the category is: strongest for connectives (**nand*) and determiners/quantifiers (**nall*, **nevery*, **noth*, **nalways*), somewhat weaker for modal auxiliaries (where *needn't* would violate the strong form of the constraint, albeit in a context in which no **E** reading would be possible), and weaker still (though still present) for ordinary adjectives (cf. *impossible* vs. *unnecessary*, where the latter but not the former is restricted to deontic, non-logical contexts).⁵

⁵ Note also the asymmetry in nominalization potential (**unnecessity* alongside *impossibility*) and in cross-linguistic parallels (Lat. **innecessarius*, Fr. **innécessaire* vs. *impossible*, *impossible*). Similar asymmetries are found in the cross-linguistic expression of modality. Tamil *veeNam* may offer an instance of a lexicalized **O** verb (de Haan 1997: 80), but the facts are more complicated than de Haan suggests (E. Annamalai, p.c.). I hope to address this issue in later work.

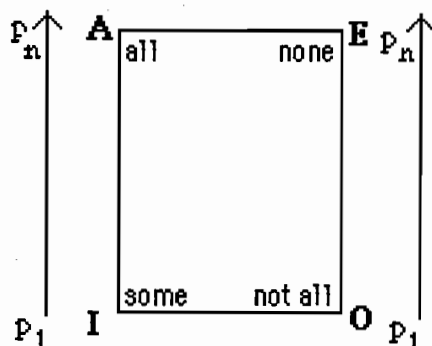
What we need for open-class categories are implicational (rather than absolute) universals. First, the existence of a lexicalized **O** form implies the existence of a lexicalized **E** counterpart but not vice versa. Second, the lexicalized **E** form tends to be more opaque and less constrained (semantically and distributionally) than the lexicalized **O** form (if any). There are also clear differences in number. For example, among English causative verbs incorporating negation, we find:

- (18) a. [CAUSE [E]]: ‘cause to become not {possible/legal/moral}’
- | | | | |
|----------|---------|-----------|-----------|
| ban | enjoin | interdict | proscribe |
| bar | exclude | preclude | refuse |
| deter | forbid | prevent | veto |
| disallow | inhibit | prohibit | withhold |
- b. [CAUSE [O]]: ‘cause to become not {necessary/obligatory}’,
‘{possible/legal/moral} not...’
- | | |
|--------|--------|
| excuse | exempt |
|--------|--------|

Among the deontics, we have simplex nominal and adjectival realizations for **A** (*obligation; obligatory/required*), **I** (*permission; permitted/allowed*), and **E** (*prohibition; forbidden*), but the **O** values can only be complex (*non-obligation, released from obligation, ...*). Among the adjectives, we do have **O** candidates like *optional*, Fr. *facultatif*, or Ger. *erübrigen*, but it’s unclear whether a particular modal (whether root, epistemic, or deontic) candidate occupies the **O** or the **Y** slot.⁶

The asymmetry of the Square also extends to “intermediate” values, south of **A/E** but north of **I/O**, where ‘not many’ can be lexicalized (= *few*) but ‘many not’ cannot, ‘not often/usually not’ can be lexicalized (= *seldom, rarely*) but ‘often not/not usually’ cannot, etc. What is the appropriate generalization? To represent the intermediate values we exploit the fact that the quantifiers, quantificational adverbs, and modal values form scales as defined by unilateral entailment (Horn 1972, following Grice 1989).

(19)

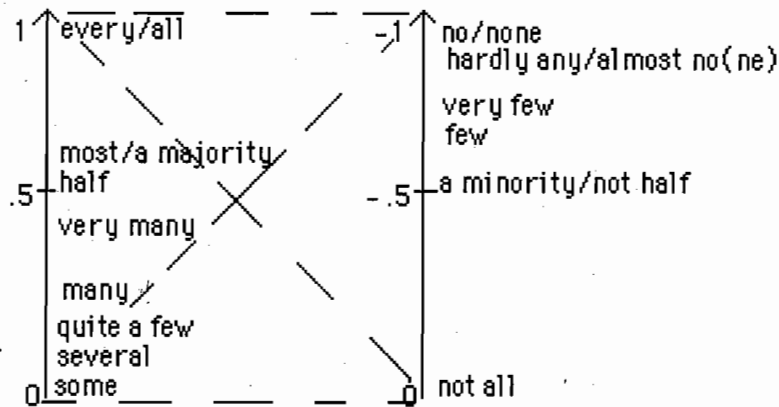


Two terms in any quantitative opposition occupy different positions on a single scale, while two terms in a qualitative opposition will occupy parallel positions (weak, intermediate, or strong) on the corresponding scale. Plotting each scalar value according

⁶ This is a more general problem. Thus, Löbner (1990: 89) nominates a number of potential instances for “Typ 4” (**O**) lexicalizations in German (cf. also Löbner 1987), but on closer inspection several of the nominees allow for other interpretations, while the viable candidates are largely limited to deontic uses.

to its lower bound in the usual way and assigning positive and negative arithmetic values to those positions, we obtain this Arithmeticized Square (Horn 1989: 236ff.)

(20)



Note that the cardinal values (A E / I O) are the strongest/weakest values on their respective scales and the sum of the values of any horizontal pair of quantificational determiners $\langle Q, Q^{-} \rangle$, whether contraries or subcontraries, is always 0.⁷

The crucial factor for determining whether an intermediate value can incorporate and inner or outer negation is where that value is situated with respect to the midpoint of its scale. This is codified by the notion of (IN)TOLERANCE (Löbner 1987: 64): “A quantifier Q is *tolerant* iff $Q(P)$ and $Q(\sim P)$ is possible at the same time. A quantifier is *intolerant* iff $Q(P)$ excludes $Q(\sim P)$ ”. Given quantity scales like those in (21),

- | | |
|---|---|
| (21) \langle all, most, many, some \rangle | \langle no/none, few/not many, not all \rangle |
| \langle always, usually, often, sometimes \rangle | \langle never, rarely/seldom, not always \rangle |
| \langle (both...) and, (either...) or \rangle | \langle neither...nor, not both \rangle |
| \langle certain, likely, possible \rangle | \langle impossible, unlikely, not certain \rangle |
| \langle must, should/ought to, can/may \rangle | \langle can't/mustn't, shouldn't, needn't \rangle |

we can assign the values to the respective positive and negative scalar positions, and obtain the generalizations in (22), as illustrated in (23):

- (22) \triangleright $\langle Q, Q^{-} \rangle$ are contraries if $Q > .5$ and subcontraries if $Q \leq .5$.
 \triangleright If $Q \leq .5$, the conjunction $Q... \text{ and } Q^{-}...$ is **consistent**, and Q is **tolerant**.
 \triangleright If $Q > .5$, the conjunction $Q... \text{ and } Q^{-}...$ is **inconsistent**, and Q is **intolerant**.

- (23) a. **Some** of my friends are linguists and **some** of them aren't.
Many of my friends are linguists and **many** of them aren't.
He **often** goes to church on Sunday and he **often** doesn't.
It's **possible** that she'll win, and **possible** that she won't.
It's **50-50** that it'll land heads, and **50-50** that it won't.
- b. **#All** of my friends are linguists and **all** of them aren't.
#Most of my friends are linguists and **most** of them aren't.
#He usually goes to church on Sunday and he **usually** doesn't.
#It's likely that it'll land heads, and **likely** that it won't.
#It's certain that she'll win, and **certain** that she won't.

⁷ For elaboration, see Horn (1989: 237) and the addenda on pp. xxxiii-xxxiv of Horn (2001).

The key principles of lexicalization (Horn 1972) can now be given as follows:

- ☞ When Q is intolerant, it may lexically incorporate its (contrary) inner negation (Q_{\neg}) but it tends not to lexicalize its outer negation ($\neg Q_T$).
- ☞ A tolerant value may incorporate its outer negation ($\neg Q_T$) but bars lexicalization of its inner negation (Q_{\neg}).

Thus, *seldom* or *rarely* lexicalize the inner negation of the intolerant *usually* (or the outer negation of the tolerant *often*), but there can be no simple lexical realization of *not usually* (or *often not*); similarly for *few*, *a minority*, or *unlikely* (see below).

The effect of these universals is to facilitate the direct expression of contrariety in natural language while delicensing the direct expression of subcontrariety. Another reflection of this contrast is the strengthening of sentential (contradictory) negation to express a contrary meaning. In the words of Bosanquet (1911: 281), “The essence of formal negation is to invest the contrary with the character of the contradictory”. The result is a variety of cases of contrary negatives in contradictory clothing (Horn 1989: Chapter 5). Some examples from English appear in (19):

(24) (a) contrary readings for affixal negation (conventionalized strengthening)

- | | |
|-----------------------------|---|
| He is unhappy | (unilaterally entails \neg [He is happy]) |
| She was unfriendly | (unilaterally entails \neg [She was friendly]) |
| I disliked the movie | (unilaterally entails \neg [I liked the movie]) |

(b) litotes/understatement in simple denials (online/non-conventionalized strengthening)

- | | |
|--|---|
| He’s not happy with it | (approaches ‘He’s unhappy with it’) |
| I don’t like ouzo | (approaches ‘I dislike ouzo’) |
| I’m not optimistic that ϕ | (approaches ‘I’m pessimistic that ϕ ’) |

(c) “neg-raising” effects (strengthened understanding as a convention of usage)

- | | |
|------------------------------------|---|
| I don’t believe it’ll snow | (\approx I believe it won’t snow) |
| I don’t want you to go | (\approx I want you not to go) |
| It’s not likely they’ll win | (\approx It’s likely they won’t win) |

In each case, the negation of an unmarked “mid-scalar” positive scalar predication (at least) implicates a stronger, contrary negation.

Consider in particular the phenomenon of “neg-raising” in (24c), a tendency first recognized by St. Anselm (1033-1109), who pointed out that “we say non debet peccare [lit., ‘s/he neg should sin’] when we mean debet non peccare” (Henry 1967: 193-94). But which predicates license lower-clause understanding of higher-clause negation? In general, neg-raising licensors are **weak intolerant** positive operators, situated just above the midpoint of the relevant scale (Horn 1989: §5.2), e.g.:

- (25) *believe, suppose* but not *know, doubt, disbelieve*
want, suggest but not *insist, forbid, prohibit*
advisable, desirable but not *obligatory, forbidden*
should, ought to, better but not *have to, must, can*
likely, probable but not *certain, impossible*
most but not *all, many, some, few*
usually but not *always, often, sometimes, rarely*

As has often been noted, neg-raising operates euphemistically, in that the expression of negative judgment is toned down, a motivation for litotes as well; in addition, as seen in the examples in (25), the weak-intolerance generalization cross-cuts the traditional distinction between “lexical” and “functional” operators.

A systematic exception to the weak intolerance is that strongly intolerant (A-position) deontic values often allow neg-raising effects. One classic example is the Fr. *Il ne faut pas que tu meures* (lit., ‘it is not necessary that you die’, but usually = ‘you mustn’t die’). This sentence is described in the eponymous paper by Tobler (1882), who notes that the notoriously “unlogisch” placement of negation was attested by 14th century alongside the “logical” O reading that it eventually comes to evict or block. Another instance of O→E drift is provided by the strengthening of negated causatives. Thus while *faire* + infinitive is normally a strong causative, its negation is often understood with an E-like strengthened meaning:

(26) *Il caffè non mi fa dormire.* ‘Coffee doesn’t {make/let} me sleep’

Not only in Italian but in Japanese, Turkish, Amharic, Czech, Biblical Hebrew, and Jacaltec, the negation of a strong causative (lit., ‘not make’) may or must strengthen to yield contrary (= ‘not let’, ‘make not’ = E) force. The reverse drift, in which a ‘not let’ (E) causative is understood as ‘let not’ or ‘not make’ (O), does not seem to be attested.

Further instances of the preferential treatment of E over O values are listed in (27), the first one having already been touched on.

- (27) a. E readings for O forms (*nealles*, Du. *nimmer*, Lat. *neque*, Russ. *nel’zja*)
 b. O→E drift for frozen forms (*not at all, pas du tout*)
 c. Opacity of E forms (*no, nary a*; Ger. *nie*; Fr. *personne, rien, jamais*)
 d. The frequent difficulty of negating A modals without subsequent drift, e.g.
 You are to leave the room. (A)
 You are not to leave the room. (E)
 e. The invariant E readings for complex adjectives with negation + poss. e.g. [un-
 [V [-able]]] = ‘incapable of being Ved’ (E), ≠ ‘capable of not being Ved’ (O)
 f. complex adverbs incorporating semantics of I (*enough*) or E (*too*) but not O
 g. the existence of “able” polarity (see (28a)) and “unable” polarity (28b) vs. the
 non-existence of “needn’t” polarity (28c); cf. Horn (1972: Chapter 3)
- (28) a. I can finally {afford a new car/tell them apart}.
 It’s possible for me to {afford a new car/tell them apart}.
 b. I couldn’t {fathom/make heads or tails of} the proposal.
 I’m incapable of {fathoming/making heads or tails of} the proposal.
 c. You needn’t *blarf* the avocado.
 It’s unnecessary (*possible/*impossible) for you to *blarf* the avocado.

Finally, we touch on one last asymmetry: when the neg-raising effect yields a strengthened E-type understanding alongside the literal O-type reading we often find that lexical incorporation of the negation renders the quasi-O readings inaccessible. Thus:

- (29) a. It’s {not probable/not likely} that a fair coin will land heads.
 (*ambiguous; true on outer [contradictory] reading of negation*)
 b. It’s {improbable/unlikely} that a fair coin will land heads.
 (*unambiguously inner [contrary] negation, hence false*)

- (30) a. It's not likely that Federer will win and not likely that he'll lose.
 (*allows tolerant, contradictory reading with outer negation*)
 b. #It's unlikely that Federer will win and unlikely that he'll lose.
 (*allows only intolerant [neg-raised], hence anomalous reading*)
- (31) a. It's not {advisable/desirable} that you go there alone. (*ambiguous*)
 b. It's {inadvisable/undesirable} that you go there alone. (*only a warning not to*)

We could also point to such minimal oppositions as *disprove* and *disbelieve* on the one hand, with $A\bar{\neg} = E$ analyses, as against *disallow* and *disable*, with $\bar{I} = E$ analyses.

The first neo-Gricean account of the asymmetries discussed in this study appeared in Horn (1972), although additional data have been accumulated since. (Cf. in particular the cross-linguistic data surveyed in Horn 1989: §4.5.) No natural language instances of **nall* or **nand* have yet surfaced. In many, probably most, languages, neither of the negative positions (**E** or **O**) is lexicalized. But the generalization remains sound: **A**, **I**, and often **E** values may lexicalize, **O** values may not. On the proposed account, the three-cornered square is a consequence of the maxim of Quantity: in any situation in which I possess (and am known to possess) complete knowledge, and in which that knowledge is (and is known to be) relevant to you, I can convey that information to you via a proposition containing one of the three values *all*, *some* (implicating *some not*, *not all*), or *none*. The fourth value (**nall* = *not all*, implicating *some*) is functionally (although not logically) expendable.

Over the succeeding decades, others have offered their own translations of the story of ***O**. Huybregts' (1979) blocking principle stipulates that *not Q* can lexicalize just when *Q not* can't, but this correlation fails to fully explain just when the latter state of affairs obtains. The monotonicity correspondence universal of Barwise & Cooper (1981: 186-87) predicts inter alia that strong determiners like *every* or *most* can incorporate inner but not outer negation; this correctly allows *neither* and *no* while ruling out **never* and **noth*. But without a grounding in the pragmatics of scalar operators, such approaches to the constraints on quantifier lexicalization are ad hoc. Worse still, any treatment based on the semantics of the determiners and quantifiers like that of Barwise & Cooper (1981) fails to generalize to the binary connectives, modals, and other non-quantificational values, much less to the intermediate values. A later proposal along similar lines is that of Hoeksema (1997), who, after reviewing earlier accounts, argues that the non-occurring **O** forms are not blocked by corresponding **I** forms, pace Horn (1972). Rather, he notes, neither of the likely sources that would yield lexicalized **O** quantifiers—the merger of an existential value + negation or the reinterpretation of NPIs—is consistent with what is known of the plausible historical development. Once again, however, the considerations he raises, contributing factors though they may be, fail to extend to other scalar values that manifest an asymmetry in lexicalization.

More recently, Seuren (2003: 13) has advanced a new formalization of the Aristotelian Predicate Calculus on which **A**, **I**, and (sometimes) **A*** (= **E**) will have lexicalized representations but **I*** (the **O** of the standard square) does not. He argues that, contra the Gricean moral to the story I have drawn, “The question [of why there is no **nall*] is superfluous...an artifact of the defective way [the Aristotelian Predicate Calculus] was formalized by Boethius” and the other geometers of the Square. But altering the representation does not explain the asymmetry in the lexical incorporation of

negation. (I must defer the more nuanced critiques of Jaspers 2005 and Seuren 2006 to another occasion.)

In sum, we draw this moral from *L'Histoire d'*O*:

The relation of mutual quantity implicature holding between positive and negative subcontraries results in the superfluity of one of the two for lexical realization, while the functional markedness of negation predicts that the unlexicalized subcontrary will always be **O** rather than **I**. The pragmatic account of the asymmetries built into the "three-cornered square" is more general and more explanatory than the rival theories that either dismiss the asymmetry as uninteresting or restrict it to the determiners and quantificational operators while ignoring other operator types along with intermediate values that can be mapped onto the Square of Opposition.

Acknowledgments

Earlier versions of some of this material were presented in Berkeley (1970), Austin (1972), Milan (2003), Lansing (2003), and New York (2004). I am indebted to those who attended those presentations, to the participants in the Montreux congress, and to Barbara Abbott, Elayaperumal Annamalai, Jack Hoeksema, Polly Jacobson, Dany Jaspers, Hans Smessaert, Rashad Ullah, Victor Sánchez Valencia, Raffaella Zanuttini, and Arnold Zwicky. Needless to say,...

References

- Aristotle. *Categories and De interpretatione*, ed. & trans. by J. L. Ackrill. Oxford: Clarendon Press, 1963.
- Ashworth, E. J. 1974. *Language and Logic in the Post-Medieval Period*. Dordrecht: Reidel.
- van der Auwera, Johan. 1996. Modality: The three-layered scalar square. *Journal of Semantics* 13: 181-96.
- van der Auwera, Johan. 2001. On the typology of negative modals. In J. Hoeksema et al. (eds.), *Perspectives on Negation and Polarity Items*, 23-48. Amsterdam: John Benjamins.
- Barwise, Jon and Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4: 159-219.
- Béziau, Jean-Yves. 2003. New light on the square of opposition and its nameless corner. *Logical Investigations* 10: 218-32.
- Blanché, Robert. 1952. Quantity, modality, and other kindred systems of categories. *Mind* 61: 369-75.
- Blanché, Robert. 1953. Sur l'opposition des concepts. *Theoria* 19: 89-130.
- Blanché, Robert. 1969. *Structures intellectuelles*, 2^e éd. Paris: J. Vrin.
- Bochenski, I. M. 1961. *A History of Formal Logic*, Ivo Thomas, trans. Notre Dame: U. of Notre Dame Press.
- Bosanquet, Bernard. 1911. *Logic*, Vol. 1, 2d edition. Oxford: Clarendon.
- Collinson, W. E. 1937. *Indication*. Language Monograph No. 17.

- Couturat, Louis. 1913. Des propositions particulières et de leur portée existentielle. *Revue de Métaphysique et de Morale*, March 1913, 256-59.
- Couturat, Louis. 1914. Réponse [to Ginzberg 1914]. *Revue de Métaphysique et de Morale*, March 1914, 259-60.
- De Morgan, Augustus. 1847. *Formal Logic*. London: Taylor & Walton.
- De Morgan, Augustus. 1858. On the syllogism: III, and on logic in general. *On the Syllogism and Other Logical Writings*, 74-146. London: Routledge & Kegan Paul.
- De Morgan, Augustus. 1861. Hamiltonian logic. *Athenæum* 13 July 1861, p. 51.
- Dewey, John. 1938. *Logic: The Theory of Inquiry*. New York: Henry Holt.
- Fogelin, Robert. 1967. *Evidence and Meaning*. New York: Humanities Press.
- Fogelin, Robert. 1978. *Understanding Arguments: An Introduction to Informal Logic*. New York: Harcourt Brace Jovanovich.
- Gazdar, Gerald and Geoffrey K. Pullum. 1976. Truth-functional connectives in natural language. *CLS* 12, 220–234, Chicago Linguistic Society, Chicago.
- Geach, P. T. 1972. *Logic Matters*. Berkeley: U. of California Press.
- Ginzberg, S. 1913. Note sur le sens équivoque des propositions particulières. *Revue de Métaphysique et de Morale*, January 1913, 101-6.
- Ginzberg, S. 1914. À propos des propositions particulières. *Revue de Métaphysique et de Morale*, March 1914, 257-59.
- Grice, H. P. 1961. The causal theory of perception. *Proceedings of the Aristotelian Society*, Supplementary Volume 35: 121-52.
- Grice, H. P. 1989. *Studies in the Way of Words*. Cambridge: Harvard U. Press.
- de Haan, Ferdinand. 1997. *The Interaction of Modality and Negation: A Typological Study*. New York: Garland.
- Hamilton, Sir William. 1860. *Lectures on Logic, Volume I*. Edinburgh: Blackwood.
- Hegenberg, Leonidas. 1957. A negação. *Revista Brasileira de Filosofia* 7: 448-57. Summarized and reviewed by Hector-Neri Castañeda, *Journal of Symbolic Logic* 25 (1960): 265.
- Henry, Desmond. 1967. *The Logic of Saint Anselm*. Oxford: Oxford U. Press.
- Hoeksema, Jack. 1999. Blocking effects and polarity sensitivity. in *JFAK: Essays Dedicated to Johan van Benthem on the Occasion of his 50th Birthday*, Vossiuspers, Amsterdam. (<http://odur.let.rug.nl/~hoeksema/docs/j50.htm>)
- Horn, Laurence. 1972. *On the Semantic Properties of Logical Operators in English*, UCLA dissertation.
- Horn, Laurence. 1973. Greek Grice: a brief survey of proto-conversational rules in the history of logic. *CLS* 9, 205–14.
- Horn, Laurence. 1989. *A Natural History of Negation*. Chicago: U. of Chicago Press.
- Horn, Laurence. 1990. Hamburgers and truth: Why Gricean inference is Gricean. *BLS* 16, 454–71.
- Horn, Laurence. 1997. All John's children are as bald as the King of France: Existential import and the geometry of opposition. *CLS* 33, 155-79. Chicago: Chicago Linguistic Society.
- Horn, Laurence. 2001. [Horn 1989], reissued with new introductory material. Stanford: Center for the Study of Language and Information.

- Horn, Laurence. 2006. The Border Wars: A neo-Gricean perspective. In K. von Heusinger and K. Turner (eds.), *Where Semantics Meets Pragmatics*, 21-48. London: Elsevier.
- Huybregts, Rini. 1979. De biologische kern van taal. In R. Huybregts and L. des Tombe (eds.), *Verkenningen in Taal*, 79-189. Utrecht: Instituut A. W. de Groot.
- Jacoby, Paul. 1950. A triangle of opposition in Aristotelian logic. *The New Scholasticism* 24. 32-56.
- Jaspers, Dany. 2005. *Operators in the Lexicon: On the Negative Logic of Natural Language*. (Universiteit Leiden dissertation.) Utrecht: LOT.
- Jespersen, Otto. 1917. *Negation in English and Other Languages*. Copenhagen: A. F. Høst.
- Jespersen, Otto. 1924. *Philosophy of Grammar*. London: Allen & Unwin. (See esp. Chap. XXIV.)
- Keynes, John Neville. 1906. *Studies and Exercises in Formal Logic*. London: Macmillan.
- Levinson, Stephen. 2000. *Presumptive Meanings: The Theory of Generalized Conversational Implicature*. Cambridge: MIT Press.
- Löbner, Sebastian. 1987. Quantification as a major module of natural language semantics. In J. Groenendijk, M. Stokhof, and D. de Jongh (eds.), *Studies in Discourse Representation Theory and the Theory of Generalized Quantifiers*, 53-85. Dordrecht: Foris.
- Löbner, Sebastian. 1990. *Wahr neben Falsch: Duale Operatoren als die Quantoren natürlicher Sprache*. Tübingen: Max Niemeyer.
- Mill, John Stuart. 1867. *An Examination of Sir William Hamilton's Philosophy* (3d edn). London: Longman.
- Monck, W. H. S. 1881. *Sir William Hamilton*. London: Sampson, Low.
- Oesterle, Jean, ed. & trans. 1962. *Aristotle: On Interpretation. Commentary by St. Thomas and Cajetan*. Milwaukee: Marquette University Press.
- Quine, Willard Van Orman. 1940. *Mathematical Logic*. New York: W. W. Norton.
- Réage, Pauline [nom de plume for Anne Desclos]. 1954. *Histoire d'O*. Sceaux: Jean-Jacques Pauvert.
- Rose, Lynn E. 1968. *Aristotle's Syllogistic*. Springfield, Ill.: Charles C. Thomas.
- Sánchez Valencia, Victor. 1994. Monotonicity in medieval logic. In *Language and Cognition* 4, 161-74. (Yearbook 1994 of the research group for theoretical and experimental linguistics of the University of Groningen.)
- Sapir, Edward. 1930. *Totality*. *Language Monograph* No. 6.
- Schaffer, Barbara. 2002. CAN'T: The negation of modal notions in ASL. *Sign Language Studies* 3: 34-53.
- Sesmat, Augustin. 1951. *Logique II*. Paris: Hermann.
- Seuren, Pieter. 2003. The logic of language: Aristotelian predicate calculus revisited. Unpublished ms., University of Nijmegen.
- Seuren, Pieter. 2006. The natural logic of language and cognition. *Pragmatics* 16: 103-38.
- Strawson, P. F. 1952. *Introduction to Logical Theory*. Methuen, London.

- Tobler, Adolf. 1882. Il ne faut pas que tu meures “du darfst nicht sterben”. *Vermischte Beiträge zur französischen Grammatik* 1, 3d ed., 201–5. Leipzig: S. Hirzel.
- Tucci, Giuseppe. 1928. On the fragments from Dīn-nāga. *Journal of the Royal Asiatic Society* 377-90, 905-06.
- Von Wright, G.H. 1951. *An Essay in Modal Logic*. Amsterdam: North-Holland.
- Whately, Richard. 1848. *Elements of Logic*, 9th edition. London: John W. Parker.
- Zabeeh, Farhang (ed. and trans.) 1971. *Avicenna's Treatise on Logic*. The Hague: Nijhoff.