Fiscal Policy and Unemployment

Abstract
This paper explores the interaction between fiscal policy and unemployment. It develops a dynamic economic model in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing government debt. In the context of this model, the paper analyzes the simultaneous determination of fiscal policy and unemployment in long run equilibrium. Outcomes with both a benevolent government and political decision-making are studied. With political decision-making, the model yields an appealing positive theory of fiscal policy and unemployment.
1 Introduction

Recent years have seen renewed interest in the use of fiscal policy to mitigate unemployment. The Great Recession revealed that politicians and policy-makers are generally optimistic about the potential fiscal policy has in this regard. Around the world, countries caught in the grip of recession pursued a variety of fiscal strategies, ranging from tax cuts to public works projects. Nonetheless, recent experience has also revealed that the willingness to use fiscal policy to aggressively fight unemployment is tempered by high levels of debt. As the Great Recession has dragged on, concern about the long-term burden of high debt has dampened political enthusiasm for further deficit-financed tax cuts and public spending hikes.

All this suggests an interesting and potentially important interaction between fiscal policy and unemployment. On the one hand, fiscal policy has the potential to mitigate unemployment. On the other, the way in which fiscal policy is used will depend on a country’s debt position which is endogenously determined by its past use of fiscal policy. This paper studies this interaction. It first constructs a dynamic economic model in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing debt. In the context of this model, the paper then analyzes the simultaneous determination of fiscal policy and unemployment in long run equilibrium. Outcomes with both a benevolent government and political decision-making are considered.

The economic model has a public and private sector. The private sector consists of entrepreneurs who hire workers to produce a private good. The public sector hires workers to produce a public good. Public production is financed by a tax on the private sector. The government can also borrow and lend in the bond market. The private sector is affected by exogenous shocks (input price hikes, financial crisis, etc) which impact private sector labor demand. Unemployment can arise because of a downwardly rigid wage. When unemployment arises, it can be mitigated by reducing taxes on the private sector and/or increasing public production. However, both actions are costly for the government.

With a benevolent government, there would be no unemployment in the long run. Moreover, the mix of public and private outputs would be optimal. The way in which a benevolent government achieves this first best outcome is by accumulating bond holdings. The earnings from these assets are used to finance unemployment mitigation when the private sector experiences negative
shocks.

The benevolent government solution is provocative in showing how governments can use fiscal policy to completely circumvent the inefficiencies stemming from labor market frictions in the long run. The important lesson suggested by the analysis is that no satisfactory theory of unemployment can abstract from how fiscal policy is chosen. Nonetheless, when interpreted as a positive theory, the solution is less interesting and this motivates introducing political decision-making. This is introduced following the approach in Battaglini and Coate (2007, 2008, 2010). Thus we assume that policy decisions are made in each period by a legislature consisting of representatives from different political districts and we introduce the friction that legislators can transfer revenues back to their districts.

With political decision-making, the model delivers an appealing positive theory of fiscal policy and unemployment. The government has no stock of assets and, when the private sector experiences negative shocks, unemployment arises. Moreover, when these shocks occur, government mitigates unemployment with debt-financed stimulus plans. These equilibrium stimulus plans typically involve both tax cuts and public production increases. When choosing such plans, the government balances the benefits of reducing unemployment with the costs of distorting the private-public output mix. In normal times, when the private sector is not experiencing negative shocks, the government reduces debt until it reaches a floor level. The existence of this floor level prevents asset accumulation as in the benevolent government solution. Unemployment can arise even in normal times, depending on the economic and political fundamentals. With or without negative shocks, when there is unemployment, it will be higher the larger the government’s debt level. High debt levels are therefore associated with high unemployment levels.

The account of the cyclical behavior of fiscal policy provided by the theory offers an alternative to the well known tax smoothing theory pioneered by Barro (1979). The tax smoothing approach suggests that governments should use budget surpluses and deficits as a buffer to prevent tax rates from changing too sharply. Thus, governments will run deficits in recessions and surpluses in booms. Underlying the approach is the assumption that the deadweight costs of taxes are a convex function of the tax rate. Our model is not a tax smoothing model in the sense that, without the labor market frictions, taxation is not distortionary. Nonetheless, the downwardly rigid wage does create distortions and the economic role for debt is to smooth these across time. Moreover, our finding that, with a benevolent government, all distortions are eliminated in the
long run parallels a similar finding for tax smoothing models.1 Furthermore, as in the model of this paper, this counter-factual long run prediction can be overcome by introducing political decision-making as shown by Battaglini and Coate (2008, 2010).

The economic model underlying our theory differs from the Keynesian and New Keynesian perspectives that underpin standard justifications for activist fiscal policy. In particular, the source of fluctuations in our economy comes from the supply rather than the demand side. In our model, recessions arise because negative shocks to the private sector reduce the demand for labor. Labor market frictions prevent the wage from adjusting and the result is unemployment. Tax cuts work by raising the return to entrepreneurs hiring labor rather than inducing consumers to spend more.2 Public spending increases are effective because the public sector increases hiring. While our modelling choices are driven by tractability rather than a belief that Keynesian demand side considerations are unimportant, we would nonetheless argue that our model captures key trade-offs that will be present in any model of activist fiscal policy. First, the model incorporates the two broad ways in which government can create jobs: indirectly by reducing taxes on the private sector, or directly through increasing public production. Second, the model captures the fact that implementing either of these strategies requires government revenue. Indirect job creation can be financed by either raising debt or by reducing public spending. However, if the latter mechanism is used, any employment losses from reducing public spending must be weighed against the benefits. Direct job creation can be financed by either raising debt or increasing taxes on the private sector. Again, if the latter mechanism is used, the employment losses from reduced private sector jobs must be weighed against the benefits. If debt is used, then this will require future reductions in public spending and/or increases in taxes on the private sector.

The organization of the remainder of the paper is as follows. Section 2 outlines the model. Section 3 studies fiscal policy and unemployment with a benevolent government. Section 4 introduces political decision-making and Section 5 concludes.

1 Under some conditions, the optimal policy in such models is for the government to gradually acquire sufficient bond holdings so as to eventually be able to finance the optimal level of public spending with the interest earnings from these holdings (Aiyagari et al (2001)). This permits the financing of government spending without distortionary taxation.

2 This is of course a key argument that is used by Republican politicians in the U.S. to justify tax cuts.
2 Model

The environment. We consider an infinite horizon economy in which there are two final goods; a private good $x$ and a public good $g$. There are two types of citizens; entrepreneurs and workers. Entrepreneurs produce the private good by combining labor $l$ and an input $z$ with their own effort. Workers are endowed with 1 unit of labor each period which they supply inelastically. The public good is produced by the government using labor.

There are $n_e$ entrepreneurs and $n_w$ workers where $n_e + n_w = 1$. Each entrepreneur produces with the Leontief production technology $x = A \min\{l, z, \epsilon\}$ where $\epsilon$ represents the entrepreneur’s effort and $A$ is a productivity parameter. The idea underlying this production technology is that when an entrepreneur hires more workers he must put in more effort to manage them. The public good production technology is $g = l$.

Workers’ per period payoff function is $x + \gamma \ln g$, where $\gamma$ measures the relative value of the public good. Entrepreneurs’ per period payoff function is $x + \gamma \ln g - \xi \epsilon^2 / 2$ where the third term represents the disutility of providing entrepreneurial effort. All individuals discount the future at rate $\beta$.

There are markets for the private good, the input, and labor. The private good is the numeraire. The input is supplied by foreign suppliers and has an exogenous but variable price $p_0$. We have in mind an input essential for production such as energy. Each period, this price can take on one of two values $p_L$ or $p_H$, where $p_L$ is less than $p_H$. We also assume throughout that $p_L$ is less than $A - \gamma / n_w$. We will say that the economy is in the low cost state when $\theta = L$ and the high cost state when $\theta = H$. The probability of the high cost state is $\alpha$. The wage is denoted $\omega$ and the labor market operates under the constraint that the wage cannot go below an exogenous minimum $\omega$. This friction is the source of unemployment. There is also a market for risk-free one period bonds. The assumption that citizens have quasi-linear utility implies that the interest rate on these bonds is $\rho = 1 / \beta - 1$.

To finance its activities, the government taxes entrepreneurs’ incomes at rate $\tau$. It can also borrow and lend in the bond market. Government debt is denoted by $b$ and new borrowing by $b'$. The government is also able to distribute surplus revenues to citizens via lump sum transfers.

Market equilibrium. At the beginning of each period, the cost state of the economy is revealed. The government repays existing debt and chooses the tax rate, public good, new borrowing, and
transfers. It does this taking into account how its policies impact the market and the need to balance its budget.

To understand how policies impact the market, assume the cost state is $\theta$ and that the tax rate is $\tau$ and the public good level is $g$. Given a wage rate $\omega$, each entrepreneur choose hiring, the input, and effort to maximize his utility

$$\max_{(l, e, z)} (1 - \tau)(A \min \{l, z, e\} - p_{gz} - \omega l) - \xi \frac{e^2}{2}.$$  \hfill (1)

Obviously, the solution involves $z = e = l$. Substituting this into the objective function and maximizing with respect to $l$ reveals that $l = (1 - \tau)(A_\theta - \omega)/\xi$ where $A_\theta = A - p_{gz}$. Aggregate labor demand from the private sector is therefore $n_c(1 - \tau)(A_\theta - \omega)/\xi$. Labor demand from the public sector is $g$ and labor supply is $n_w$. Setting demand equal to supply, reveals that the market clearing wage is

$$\omega = A_\theta - \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right).$$  \hfill (2)

The minimum wage will bind if this wage is less than $\omega$. In this case, the equilibrium wage is $\omega$ and the unemployment rate is

$$u = \frac{n_w - g - n_c(1 - \tau)(A_\theta - \omega)/\xi}{n_w}.$$  \hfill (3)

To sum up, in state $\theta$ with government policies $\tau$ and $g$, the equilibrium wage rate is

$$\omega_\theta = \begin{cases} 
\omega & \text{if } A_\theta \leq \omega + \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right) \\
A_\theta - \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right) & \text{if } A_\theta > \omega + \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right)
\end{cases}$$  \hfill (4)

and the unemployment rate is

$$u_\theta = \begin{cases} 
\frac{n_w - g - n_c(1 - \tau)(A_\theta - \omega)/\xi}{n_w} & \text{if } A_\theta \leq \omega + \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right) \\
0 & \text{if } A_\theta > \omega + \xi \left( \frac{n_w - g}{n_c(1 - \tau)} \right).
\end{cases}$$  \hfill (5)

When the minimum wage is binding, the unemployment rate is increasing in $\tau$. Higher taxes cause entrepreneurs to put in less effort and this reduces private sector demand for workers. The unemployment rate is also decreasing in $g$ because to produce more public goods, the government must hire more workers. When the minimum wage is not binding, the equilibrium wage is decreasing in $\tau$ and increasing in $g$. 

5
Each entrepreneur earns profits of \( \pi_\theta = (1 - \tau)(A_\theta - \omega_\theta)^2 / \xi \). Assuming he receives no government transfers and consumes his profits, an entrepreneur obtains a period payoff of
\[
v_{\pi_\theta}(\tau, g) = \frac{(A_\theta - \omega_\theta)^2(1 - \tau)^2}{2\xi} + \gamma \ln g.
\] (6)

Jobs are randomly allocated among workers and so each worker obtains an expected period payoff
\[
v_{\omega_\theta}(\tau, g) = (1 - w_\theta)\omega_\theta + \gamma \ln g.
\] (7)

Again, this assumes that the worker receives no transfers and simply consumes his earnings.

Aggregate output of the private good is \( x_\theta = n_\theta A(1 - \tau)(A_\theta - \omega_\theta) / \xi \). Substituting in the expression for the equilibrium wage, we see that
\[
x_\theta = \begin{cases} 
n_\theta A(1 - \tau)(A_\theta - \omega) / \xi & \text{if } A_\theta \leq \omega + \xi\left(\frac{n - g}{n(1 - \tau)}\right) \\
A(n_\omega - g) & \text{if } A_\theta > \omega + \xi\left(\frac{n - g}{n(1 - \tau)}\right) \end{cases}.
\] (8)

Note that the tax rate has no impact on private sector output when the minimum wage constraint is not binding. This is because labor is inelastically supplied and as a consequence the wage adjusts to ensure full employment. A higher tax rate just leads to an offsetting reduction in the wage rate. However, when there is unemployment, tax hikes reduce private sector output because they lead entrepreneurs to reduce effort. Public good production has no effect on private output when there is unemployment, but reduces it when there is full employment.

The government budget constraint  Having understood how markets respond to government policies, we can now formalize the government’s budget constraint. Tax revenue is
\[
R_\theta(\tau, \omega_\theta) = \tau(n_\theta \pi_\theta) = \tau n_\theta(1 - \tau)(A_\theta - \omega_\theta)^2 / \xi.
\] (9)

Total government revenue is therefore \( R_\theta(\tau, \omega_\theta) + b' \). The cost of public good provision and debt repayment is \( \omega g + b(1 + \rho) \). The budget surplus available for transfers is the difference between \( R_\theta(\tau, \omega_\theta) + b' \) and \( \omega g + b(1 + \rho) \). The government budget constraint is that this budget surplus be non-negative, which requires that
\[
R_\theta(\tau, \omega_\theta) - \omega g g \geq b(1 + \rho) - b'.
\] (10)

There is also an upper limit \( b \) on the amount of debt the government can issue. This limit is motivated by the unwillingness of borrowers to hold bonds that they know will not be repaid.
If, in steady state, the government were borrowing an amount $b$ such that the interest payments exceeded the maximum possible tax revenues in the high cost state; i.e., $\rho b > \max_\tau R_H(\tau, \omega)$, then, if the economy were in the high cost state, it would be unable to repay the debt even if it provided no public goods or transfers. Thus, the upper limit on debt is $\bar{b} = \max_\tau R_H(\tau, \omega)/\rho$.

3 Benevolent government

It will prove instructive to break down the analysis of the benevolent government’s solution into two parts. First, we study the static optimal policy problem for this economy. Thus, we ignore debt and, in the spirit of the optimal taxation literature, assume that the government faces an exogenous revenue requirement. Having understood how the static solution depends on the revenue requirement, we then introduce debt and study the dynamic policy choice problem. In the dynamic problem, the government’s revenue requirement corresponds to the difference between debt repayment and new borrowing (as in (10)). Solving the dynamic model endogenizes the government’s revenue requirement and completes the picture of the solution.

3.1 The static problem

The static optimal policy problem is to choose a tax rate $\tau$ and a level of public good $g$ to maximize aggregate citizen utility subject to the requirement that revenues net of public production costs cover a revenue requirement $r$. To allow for the possibility of surpluses or deficits when debt is introduced, we assume that the revenue requirement can be positive or negative. Under the assumption that any surplus revenues are transferred to the citizens, this problem can be posed as:

$$\max_{(\tau, g)} \left\{ R_\theta(\tau, \omega) - \omega g - r + n_\theta v_{\theta \theta}(\tau, g) + n_\omega v_{\omega \omega}(\tau, g) \right\} \quad \text{s.t. } R_\theta(\tau, \omega) - \omega g \geq r$$

(11)

What makes this problem non-standard is the endogenous wage and the possibility of unemployment. The problem of handling the endogenous wage can be simplified by noting that there is no loss of generality in assuming that the government always sets taxes sufficiently high so that the equilibrium wage equals $\omega$. As noted earlier, taxes are non-distortionary when the wage exceeds $\omega$ and the government has the ability to make transfers. Thus, if the wage exceeded $\omega$, there would be no change in aggregate utility if the government raised taxes and simply redistributed the additional tax revenues back to the citizens. This observation allows us to write problem (11)
as:

$$\max_{(\tau, g)} \left\{ x_{\theta}(\tau) \left( \frac{A^x}{A} \right) - n_{\epsilon} \xi \left( \frac{x_{\theta}(\tau)}{A_{\theta}} \right)^2 + \gamma \ln g - r \right\},$$

s.t. $R_{\theta}(\tau, \omega) - \omega g \geq r \& g + \frac{x_{\theta}(\tau)}{A} \leq n_{w}$ \hspace{1cm} (12)

where $x_{\theta}(\tau)$ is the output of the private good when the tax rate is $\tau$ and the wage rate is $\omega$ (see (8)).

Problem (12) has a simple interpretation. The objective function is the aggregate surplus generated by outputs $x_{\theta}(\tau)$ and $g$, less the revenue requirement. The aggregate surplus expression reflects the fact that, given labor is supplied inelastically, the only costs of producing the private good are those associated with the input and entrepreneurial effort. The first constraint is the government budget constraint under the assumption that the wage is $\omega$. The second constraint ensures that the equilibrium wage is $\omega$. Given the policies $(\tau, g)$, private sector demand for labor at wage $\omega$ is $x_{\theta}(\tau)/A$ and public sector demand is $g$. The constraint therefore requires that the demand for labor at wage $\omega$ is less than or equal to the number of workers $n_{w}$. With the wage fixed at $\omega$, it can be thought of as the economy's resource constraint.

We will use a diagrammatic approach to characterize the solution to problem (12). To ensure that the problem has a solution, we assume that $r$ is less than or equal to the maximum possible tax revenue which is $\max_{\tau} R_{\theta}(\tau, \omega)$.\footnote{The revenue maximizing rate is $r = 1/2$ and thus the maximum revenue requirement is $n_{\epsilon} A_{\theta} (A_{\theta} - \omega) / 4 \xi$.} We also assume that unemployment would result if the government faced the maximal revenue requirement.\footnote{This assumption amounts to the requirement that $n_{w}$ exceeds $n_{\epsilon} (A_{\theta} - \omega) / 2 \xi$.} To understand our diagrammatic approach, consider Fig 1.A. The tax rate is measured on the horizontal axis and the public good on the vertical. The upward sloping line is the resource constraint. Using the expression for $x_{\theta}(\tau)$ from (8), this line is described by

$$g = n_{w} - n_{\epsilon} (1 - \tau)(A_{\theta} - \omega) / \xi.$$  

At points along this line, there is full employment at the wage $\omega$. Policies must be on or below this line and points below are associated with unemployment.

The upward sloping, convex curves represent the government's indifference curves. These curves tell us the government's preferences over different $(\tau, g)$ pairs. Indifference curves satisfy...
Figure 1: Activist fiscal policy: the static case.
for some target utility level $U$

$$x_0(r) \left( \frac{A_\theta}{A} \right) - n_c \xi \left( \frac{z_0(\tau)}{A_\theta n_c} \right)^2 + \gamma \ln g = U. \tag{14}$$

Higher indifference curves are associated with higher utility levels, so utility is increasing as we move North-West. The indifference curves become flatter as we move South-East and the public good becomes more scarce.

The tangency point between the indifference curves and the full employment line defines the first best policies $(\tau_\theta^*, g_\theta^*)$. When these policies are in place, there is both full employment at wage $\omega$ and the optimal mix of private and public outputs. It is straightforward to show that the optimal public good level in state $\theta$ is

$$g_\theta^* = \sqrt{2 (A_\theta n_c - \xi n_w)^2 + 4 \xi n_c \gamma - (A_\theta n_c - \xi n_w)} - \frac{(A_\theta n_c - \xi n_w)}{2\xi}. \tag{15}$$

The associated tax rate $\tau_\theta^*$ provides entrepreneurs with just the right incentive to employ those workers not employed in the public sector at the wage rate $\omega$ and is given by

$$\tau_\theta^* = 1 - \frac{\xi (n_w - g_\theta^*)}{n_c (A_\theta - \omega)}. \tag{16}$$

In the remaining panels of Figure 1, we add the government’s budget line - the locus of points that satisfy the budget constraint with equality. The budget line associated with revenue requirement $r$ can be solved to yield

$$g = \frac{R_\theta(\tau, \omega)}{\omega} - \frac{r}{\omega}. \tag{17}$$

Policies must be on or below this line and points below are associated with positive transfers. Each budget line is hump shaped, with peak at $\tau = 1/2$. Increasing the revenue requirement shifts down the budget line but does not change the slope. Panels B, C, and D of Fig 1 represent increasing revenue requirements. The feasible set of $(\tau, g)$ pairs for the optimal taxation problem are those that lie below both the budget and resource constraints (represented by the gray areas in Figure 1). This forms a (weakly) convex set and so the problem is well-behaved.

We can now characterize the optimal policies. Three cases may be distinguished.

**Case 1: Full Employment with No Distortions.** In Panel B, the revenue requirement is small enough so that the first best point $(\tau_\theta^*, g_\theta^*)$ lies in the feasible set. The government can therefore select this and have revenue left over to rebate back to the citizens via a positive
transfer. In this case, the budget constraint is not binding. This case arises when \( r \) is less than 
\[ r^*_0 = R_0(\tau^*_0, \omega) - \omega g^*_0. \]

**Case 2: Full Employment with Distortions.** In this case, the government distorts taxes and public production so as to achieve full employment. To be in this case, the budget line must lie above the resource constraint for some range of taxes. When it does, there will be a range of policies that can achieve full employment. In Panel C, for example, the government can achieve full employment by choosing any tax rate in the range \([\tau^-_0(r), \tau^+_0(r)]\) with associated level of public good given by (13). As \( r \) increases, this set shrinks both on the right and on the left (i.e., \( \tau^+_0(r) - \tau^-_0(r) \to 0 \)).

If the government does choose full employment, it will choose the tax rate \( \tau^-_0(r) \) with associated public good level \( g^-_0(r) \) if \( \tau^*_0 \) is to the left of \( \tau^-_0(r) \). This is because \( (\tau^-_0(r), g^-_0(r)) \) is the closest point on the full employment line to the first best point \( (\tau^*_0, g^*_0) \). By similar logic, if \( \tau^*_0 \) is to the right of \( \tau^+_0(r) \), the government will choose the tax rate \( \tau^+_0(r) \) with associated public good level \( g^+_0(r) \). The former case is illustrated in Fig 1.C and Fig 2.A, the latter in Fig 2.B. In the former case, the government maintains full employment by raising taxes and public production. In the latter, it does so by reducing taxes and public production. In the former case, the government distorts the output mix towards public production and in the latter it distorts away from public production. These cases also generate different comparative static implications. In the former case, as the revenue requirement increases, taxes and public production increase, as illustrated in Fig 2.A. In the latter, taxes and public production decrease as illustrated in Fig 2.B.

It is notable that even in such a simple model, the direction of activist fiscal policy depends upon the underlying details of the economy. In the Appendix, we show that \( \tau^*_0 \) is to the left of \( \tau^-_0(r) \) if and only if:

\[
\gamma < \frac{A_0}{2} \left[ \frac{n_\omega - n_o A_0}{2 \xi} \right]. \tag{18}
\]

Thus, this condition determines whether the government increases or reduces the size of government to achieve full employment. Intuitively, when \( \gamma \) is small, both taxes and public production are small in the unconstrained solution. This means that raising taxes will have a relatively small impact on private sector employment. Thus, when a higher revenue requirement must be met, a fiscal surplus can be created by taxing the private sector and hiring the displaced workers in the public sector.
Case 3: Unemployment. In this case, the optimal policies involve unemployment. A sufficient condition for this case is that the budget line lies everywhere below the resource constraint as in Panel D. In this scenario, it is impossible for the government to maintain full employment. However, even when the government can maintain full employment, it will choose not to do so if the necessary distortions required in the output mix outweigh the benefits of full utilization of resources.

In fact, there will be a critical value of the revenue requirement below which the government will choose unemployment. Let \((\tilde{r}_0(r), \tilde{g}_0(r))\) denote the point at which the indifference curve is tangent to the budget line when the revenue requirement is \(r\). As \(r\) decreases, the tax rate decreases and the public production level increases, so \((\tilde{r}_0(r), \tilde{g}_0(r))\) moves to the North-West. Unemployment also decreases. Let \(r_0^{**}\) denote the net borrowing level at which \((\tilde{r}_0(r), \tilde{g}_0(r))\) lies on the full employment line and let \((r_0^{**}, g_0^{**}) = (\tilde{r}_0(r_0^{**}), \tilde{g}_0(r_0^{**}))\). Then, when the revenue requirement is greater than \(r_0^{**}\), unemployment will result and the optimal policy will equal \((\tilde{r}_0(r), \tilde{g}_0(r))\). When the revenue requirement is less than \(r_0^{**}\), the government will choose full employment.

The fact that the government will not necessarily choose to maintain full employment is illus-

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5 The slope of the budget line is \(n_c(1 - 2\tau)(A_\theta - \omega)^2/\xi\omega\) and the slope of the indifference curves is \(n_c(A_\theta - \omega)[\tau(A_\theta - \omega) + \omega]|\gamma/\xi\). Equating the two, we find that \(g\) must equal \(\gamma(1 - 2\tau)(A_\theta - \omega)/\xi\omega(\tau(A_\theta - \omega) + \omega)\). Combining this with (17) yields two equations which can be solved for \((\tilde{r}_0(r), \tilde{g}_0(r))\).
trative of an important general lesson: the government will trade off minimizing unemployment with distorting the mix of public and private outputs. When full employment cannot be achieved, the unemployment minimizing tax rate is \( \tau_0^* = (A_\theta - 2\omega)/2(A_\theta - \omega) \) with associated public good level \( g_0^*(r) \) given by (17).\footnote{This assumes that \( g_0^*(r) = \frac{R_\theta(\tau_0^*, \omega)}{\omega} - \frac{r}{\omega} \) is non-negative. If it is negative, then the unemployment minimizing tax rate is such that \( R_\theta(r, \omega) = r \) and the associated public good level is 0.} This is the point at which the slope of the budget line is equal to the slope of the full employment line. The optimal policy choice \((\tilde{\tau}_0(r), \tilde{g}_0(r))\) will not in general equal \((\tau_0^*, g_0^*(r))\), so that unemployment will be higher than it needs to be. The optimal policy could involve a lower or higher tax rate than the unemployment minimizing rate depending upon the parameters of the economy and the size of the revenue requirement.\footnote{As noted above, for sufficiently large \( r \), the unemployment minimizing tax rate is such that \( R_\theta(r, \omega) = r \). However, the government will choose to provide some public good for any \( r \) less than the maximum level, implying that the tax rate exceeds the unemployment minimizing level.} When it involves a lower tax rate, increasing the size of government would create jobs but the government holds back because the lost private output is more valuable than the additional public output. When the optimal policy involves a higher tax rate, reducing the size of government would create jobs but the government holds back for the opposite reason.

Putting all this together, we have the following characterization of the solution to problem (12).

**Proposition 1** The solution to problem (12) has the following properties.

- If \( r \leq \tau_0^* \), the solution involves full employment with no distortions. The optimal policies are \((\tau_0^*, g_0^*)\) and are independent of the revenue requirement. In this range, an increase in \( r \) is absorbed by a reduction in government transfers.

- If \( r \in (\tau_0^*, \tau_0^{**}] \), the solution involves full employment with distortions. If (18) is satisfied, the optimal policies are \((\tau_0^-(r), g_0^-(r))\) and the output mix is distorted in favor of the public good. In this range, as \( r \) increases both public production and the tax rate increase. If (18) is not satisfied, the optimal policies are \((\tau_0^+(r), g_0^+(r))\) and the output mix is distorted in favor of the private good. As \( r \) increases, both public production and the tax rate decrease.

- If \( r > \tau_0^{**} \), the solution involves unemployment. The optimal policies are \((\tilde{\tau}_0(r), \tilde{g}_0(r))\) and in general will differ from the unemployment minimizing policies. In this range, as \( r \) increases public production decreases, the tax rate increases, and unemployment increases.
It is important to note that the solution described in Proposition 1 reflects our (w.l.o.g.) assumption that the government sets a tax rate such that the wage is \( \omega \). When \( r \) is less than \( r_\theta \), the government could equally well reduce the tax rate and let the wage rate rise above \( \omega \), compensating for the lost tax revenues by reducing transfers. Thus, in the case of full employment with no distortions, the optimal tax rate and level of transfers are not uniquely defined. In all the other cases, the solution must be exactly as described in Proposition 1.

Proposition 1 tells us how taxes, public good production, and employment in each state depend on the government's revenue requirement. A premise of the analysis is that the revenue requirement is exogenous. In a dynamic model, however, \( r \) is endogenous, depending on the amount of government debt that needs to be repaid and new borrowing.\(^8\) Proposition 1, therefore, leaves a key question unanswered. In which of the three cases listed above should we expect the government to be in the long run?

### 3.2 Dynamics

With this understanding of optimal taxation and public production, we now bring debt into the picture. Debt will be helpful since it enables the government to smooth distortions across periods. The dynamic problem is to choose a time path of policies to maximize aggregate lifetime citizen utility. The problem can be formulated recursively as

\[
V_\theta(b) = \max_{(\tau, g, b', b, w_\theta)} \left\{ B_\theta(\tau, g, b', b, w_\theta) + n_w v_{w_\theta}(\tau, g) + n_w v_{w_\theta}(\tau, g) + \beta EV_\theta(b') \right\},
\]

\[s.t. \ B_\theta(\tau, g, b', b, w_\theta) \geq 0, \ b' \leq \bar{b}\]

(19)

where \( V_\theta(b) \) is aggregate lifetime citizen utility in state \( \theta \) with initial debt level \( b \) and \( B_\theta(\cdot) \) denotes the budget surplus available for transfers.\(^9\) Thus, given the state of the economy \( \theta \) and an initial debt level \( b \), the government chooses the current tax rate \( \tau \), the public good level \( g \), and new borrowing \( b' \). Transfers are determined residually by \( B_\theta(\tau, g, b', b, w_\theta) \).

As in the static problem, there is no loss of generality in assuming that the government always sets taxes sufficiently high that the wage is equal to \( w \). Thus, proceeding as in the static case and

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\(^8\) Specifically, from (10), we see that \( r \) will equal \( (1 + \rho)b - b' \).

\(^9\) That is, \( B_\theta(\tau, g, b', b, w_\theta) = R_\theta(\tau, w_\theta) + b' - \omega_\theta g - b(1 + \rho) \).
substituting \( b(1 + \rho) - b' \) for \( r \), we may rewrite the government’s problem as

\[
V_\theta(b) = \max_{(r, g, b')} \left\{ b' - b(1 + \rho) + x_\theta(r) \left( \frac{A_0}{A} \right) - n_e \xi \left( \frac{z_\theta(r)}{n_w} \right)^2 + \gamma \ln g + \beta E V_\theta(b') \right\}.
\]

s.t. \( B_\theta(r, g, b', b, w) \geq 0, g + \frac{z_\theta(r)}{A} \leq n_w, b' \leq b \) \( (20) \)

We impose an assumption to focus the analysis on the natural case of interest. Specifically, we assume that when debt is zero the government is able to achieve the first best without borrowing in the low but not the high cost state. More precisely, we make:\(^{10}\)

**Assumption 1**

\[
R_H(r^*_H, w) - wg^*_H < 0 < R_L(r^*_L, w) - wg^*_L.
\]

Recalling the definition of \( r^*_\theta \), the critical revenue requirement in Proposition 1 delineating Cases 1 and 2, Assumption 1 implies that \( r^*_H \) is negative and \( r^*_L \) is positive.

A solution to problem (20) consists of optimal policy functions \( \{r_\theta(b), g_\theta(b), b'_\theta(b)\} \) for each state \( \theta \) and value functions \( V_H(b) \) and \( V_L(b) \). By standard methods, it can be shown that there exists a solution. Corresponding to any solution, we can define \( r_\theta(b) = (1 + \rho)b - b'_\theta(b) \) to be the revenue requirement implied by the optimal policies in state \( \theta \) with initial debt level \( b \).

Letting \( (r^*_\theta(r), g^*_\theta(r)) \) denote the optimal static policies described in Proposition 1, it is clear that \( (r_\theta(b), g_\theta(b)) \) will equal \( (r^*_\theta(r_\theta(b)), g^*_\theta(r_\theta(b))) \). As discussed above, therefore, the key issue is to identify how the revenue requirement behaves in the long run. This will tell us which of the three cases described in Proposition 1 will arise.

To study the long run, note that given a solution to problem (20), for any initial debt level \( b_0 \) and sequence of shocks \( \{ \theta_t \} \), we can associate a sequence of policies \( \{r_t, g_t, b'_t\} \).\(^{11}\) The associated sequence of revenue requirements is then \( \{r_t\} \) where for all \( t, r_t = (1 + \rho)b'_{t-1} - b'_t \). The question is how these sequences behave as \( t \) becomes large. In fact, we can show that the probability that \( r_t \) is less than or equal to \( r^*_\theta \) converges to one as \( t \) becomes large. From Proposition 1, we may conclude that, in the long run, the relevant case is the first. Thus we have:

\(^{10}\) In the Appendix, we show that in terms of the fundamental parameters of the model this assumption amounts to:

\[ n_e A_L - \xi n_w < n_e \left( \frac{\gamma}{n_w} + \frac{\xi \gamma}{n_w} \right) < n_e A_H - \xi n_w. \]

\(^{11}\) This sequence is defined inductively as follows: \( \{r_0, g_0, b'_0\} = \{r_0(b_0), g_0(b_0), b'_0(b_0)\} \) and for all \( t \geq 1, \)

\( \{r_t, g_t, b'_t\} = \{r_t(b_{t-1}), g_t(b_{t-1}), b'_t(b_{t-1})\}. \)

15
Proposition 2 In any solution to problem (20) the economy converges to full employment with no distortions.

In the long run, therefore, in state $t$, taxes and public production are $(\tau^*_t, g^*_t)$. In the high cost state ($\theta = H$) public production is higher and tax revenues are lower. The increase in public production occurs because, while the benefit of public goods is state independent, the cost of the private good is higher. Lower tax revenues also reflect the fact that the private sector is less profitable.\footnote{The impact on the tax rate of moving from the low cost to the high cost state is ambiguous. On one hand, to hire any given number of workers, entrepreneurs need to be provided with lower taxes in the high cost state since workers are less profitable. On the other hand, entrepreneurs need to hire fewer workers because public production increases.} Despite lower net tax revenues, the government is able to implement the first best policies in the high cost state in the long run because it is has accumulated sufficient bond holdings.

Precisely how the government finances its activities is not tied down by the theory because there are multiple solutions to problem (20) and financing will depend on the details of the solution. The simplest solution is that the government gradually accumulates bonds until its debt level reaches $r^*_H/\rho$ (recall that $r^*_H$ is negative by Assumption 1). Once debt reaches this level, the steady state revenue requirement is $r^*_H$. This negative revenue requirement reflects the fact that the government is earning interest on its bond holdings. In the high cost state, these interest earnings are just sufficient to finance the shortfall in net tax revenues. In the low cost state, these interest earnings are rebated back to the citizens in a transfer along with the surplus net tax revenues $r^*_L$.\footnote{As noted following Proposition 1, in the low cost state, the government could equally well reduce the tax rate and let the wage rate rise above $\omega$, compensating for the lost tax revenues by reducing transfers. In this case, we would observe wage reductions rather than transfer reductions when the economy moves from the low to the high cost state.} Intuitively, other solutions are possible because once debt has reached $r^*_H/\rho$ the government can further reduce it temporarily with no effect on citizens' utility.

Proposition 2 strikes us as interesting from a normative perspective. It suggests that, in the presence of labor market frictions, there is an intimate connection between fiscal policy and unemployment. In the model developed here, in the long run a benevolent government employs fiscal policy to circumvent the inefficiencies and achieve full employment with no distortions in all states. The general lesson hinted at is that no satisfactory theory of unemployment can abstract from how fiscal policy is chosen. Nonetheless, when interpreted as a positive theory of fiscal policy and unemployment, this benevolent government's solution is obviously unsatisfactory. This leads
us to introduce political decision-making into the model.

4 Political decision-making

We now study fiscal policy and unemployment with political decision-making. Our modeling strategy follows Battaglini and Coate (2007, 2008, 2010). Thus, we assume that the economy is divided into $N$ identically sized political districts, each a microcosm of the economy as a whole. In each period, policy decisions are made by a legislature consisting of $N$ representatives, one from each district. The budget surplus $B_\theta$ can be divided among the districts in any way the representatives choose. Legislative decisions are made by a process in which representatives are randomly selected to make proposals to the floor. The votes of $Q < N$ representatives are required to pass any proposal. If no proposal has been passed after a specified number of proposal rounds, one representative is randomly picked to choose a policy. This representative is required to choose a policy that divides the budget surplus evenly between districts. Each representative's payoff is assumed to equal the aggregate utility of citizens in his district.

In this set-up, in state $\theta$ with initial debt level $b$, the equilibrium policies $\{\tau_\theta(b), g_\theta(b), b_\theta(b)\}$ solve the maximization problem

$$\max_{\{\tau, g, b\}} \left\{ q B_\theta(\tau, g, b, b, \omega_\theta) + n_e v_{e\theta}(\tau, g) + n_w v_{w\theta}(\tau, g) + \beta [\alpha V_H(b') + (1 - \alpha) V_L(b')] \right\}$$

s.t. $B_\theta(\tau, g, b', b, \omega_\theta) \geq 0, b' \leq b$. \quad (21)

where $q = N/Q$ and $V_H(b)$ and $V_L(b)$ denote the representatives' equilibrium value functions. These value functions $V_H(b)$ and $V_L(b)$ are defined recursively by the equations

$$V_\theta(b) = \begin{cases} B_\theta(\tau_\theta(b), g_\theta(b), b_\theta(b), b, \omega_\theta) + n_e v_{e\theta}(\tau, g) + n_w v_{w\theta}(\tau, g) \\ + \beta [\alpha V_H(b_\theta(b)) + (1 - \alpha) V_L(b_\theta(b))] \end{cases} \quad \forall \theta \in \{L, H\}. \quad (22)$$

The easiest way to understand the equilibrium intuitively is to imagine that in each period a minimum winning coalition of $Q$ representatives is randomly chosen and that this coalition chooses policy to maximize its aggregate utility.\footnote{We stress that we are not assuming that this is what happens but the outcome of the equilibrium is \textit{as if} this happens.} Problem (21) reflects the coalition's maximization problem and, because membership in this coalition is random, all representatives are ex ante identical and have a common value function given by (22). Given what we have assumed about
representatives' payoffs, this common value function is also a measure of aggregate citizen lifetime utility.

Formally, a political equilibrium consists of policy functions \( \{ \tau_0(b), g_0(b), b_0(b) \} \) for each state \( \theta \) and value functions \( V_H(b) \) and \( V_L(b) \) such that (i) the policy functions solve (21) given the value functions, and, (ii) the value functions satisfy (22) given the policy functions. An equilibrium is said to be well-behaved if the associated value functions \( V_H(b) \) and \( V_L(b) \) are concave for debt levels below \( \bar{b} \). In the Appendix, we show:

**Proposition 3** There exists a well-behaved equilibrium.

The equilibrium policies can be characterized by solving problem (21). Again, there is no loss of generality in assuming that the minimum winning coalition (mwc) will always set taxes sufficiently high that the wage is \( \omega \). Indeed, because \( q > 1 \), it must be the case that the wage is \( \omega \), for the mwc would always raise taxes if it could extract more revenue with no deadweight cost. Thus, we can rewrite (21) as:

\[
\max_{(\tau, g, b')} \left\{ \begin{array}{c}
\frac{x_0(\tau) \left( \frac{A_{\text{eff}}}{A} \right)}{2} - n g - \frac{q(r(\tau))}{A_{\text{eff}}} - \frac{\gamma \ln g}{2} + \beta \left( \alpha V_H(b') + (1 - \alpha) V_L(b') \right) \\
+ (q - 1) (R_0(\tau, \omega) - \omega g) + q (b' - (1 + \rho)b)
\end{array} \right\}
\]

\[
\text{s.t. } B_0(\tau, g, b', \omega) \geq 0, \quad g + \frac{x_0(\tau)}{A} \leq n_w, \quad b' \leq \bar{b}.
\]

(23)

To understand the equilibrium policies we follow the procedure used for the benevolent government case. First, we investigate the equilibrium tax and public good levels for a given revenue requirement. Then we understand the revenue requirements that arise in steady state by characterizing the equilibrium debt distribution.

### 4.1 The static problem

The equilibrium version of static problem (12) is

\[
\max_{(\tau, g)} \left\{ \begin{array}{c}
\frac{x_0(\tau) \left( \frac{A_{\text{eff}}}{A} \right)}{2} - n g - \frac{q(r(\tau))}{A_{\text{eff}}} - \frac{\gamma \ln g - q r + (q - 1) (R_0(\tau, \omega) - \omega g)}{2} \\
\end{array} \right\}
\]

\[
\text{s.t. } R_0(\tau, \omega) - \omega g \geq r & \text{ & } g + \frac{x_0(\tau)}{A} \leq n_w
\]

(24)

The key difference between this and problem (12) is that, since \( q > 1 \), the politicians put more weight on tax revenues net of public production costs, \( R_0(\tau, \omega) - \omega g \). This implies that the indifference curves associated with the objective function (24) are steeper in \((\tau, g)\) space.
There are two possible cases to consider, depending on the size of $q$. This variable, which measures (inversely) the fraction of the legislature required to pass legislation, turns out to play a key role in determining the nature of the solution.

4.1.1 Low $q$

Following the strategy used to study the static problem, consider what happens when the revenue requirement is so low that the budget constraint is not binding. In the case represented in Fig 3.A, the mwc's optimal choice $(\tau^*_q, g^*_q)$ corresponds to the point of tangency between the indifference curve and the resource constraint. This can be solved to yield:

$$(\tau^*_q, g^*_q) = \left(1 - \frac{\xi}{n_e(A_\theta - \omega)} \sqrt{\frac{(q A_\theta n_e - (2q - 1) \xi n_w)^2 + (2q - 1) 4 \xi n_e \gamma - (q A_\theta n_e - (2q - 1) \xi n_w)}{2q - 1) 2 \xi}} \right)$$

This case is similar to the situation illustrated in Fig 1.B, although now the indifference curves are steeper, so $g^*_q$ is smaller than $g^*_w$.

Define $r^*_q$ to be the revenue requirement equal to $R_\theta(\tau^*_q, \omega) - \omega g^*_q$. Then, if the revenue requirement is less than or equal to $r^*_q$ the mwc will choose the optimal tax and public production levels (25). Even though we have full employment and distortions in this case, the reason for the
distortions is not to create jobs but to extract revenue for transfers. Thus, we refer to this case as full employment with optimal revenue extraction.

If \( r > r_g^* \) the mwc will not make transfers to their districts and the budget constraint will bind. The optimal policy will then solve:

\[
\max_{(\tau, \phi)} \left\{ \frac{z_\phi(\tau)}{A} \left( \frac{A - 1}{A} \right) - n_c \zeta \left( \frac{z_\phi(\tau)}{A} \right)^2 + \gamma \ln g - qr \right\},
\text{s.t. } R_\phi(\tau, \omega) - \omega g \geq r & \& g + \frac{z_\phi(\tau)}{A} \leq n_w.
\]

This is equivalent to the problem studied in Section 3 and thus the solution will be as described by Proposition 1.\(^{15}\) There will be two regions, one with full employment with distortions (\( r \leq r_g^* \)) and one with unemployment (\( r > r_g^* \)).

The case represented in Fig ?? arises when the optimal policy ignoring the budget constraint is at the tangency point with the resource constraint. We show in the appendix that this occurs if and only if \( q \) is less than \( q_g^* \), where \( q_g^* \) is defined by:

\[
n_r \left[ \frac{q(\Lambda - \omega) + \omega}{\xi(2q - 1)} \right] + \frac{\gamma}{(q - 1) \omega} = n_w.\]

The analogy to Proposition 1 in this case is therefore:

**Proposition 4** When \( q < q_g^* \) the solution to problem (24) has the following properties.

- If \( r \leq r_g^* \), the solution involves full employment with optimal revenue extraction. The equilibrium policies are \((\tau_g^*, \phi_g^*)\) and are independent of the revenue requirement.

- If \( r \in (r_g^*, r_g^{**}] \), the solution involves full employment with distortions. If (18) is satisfied, the equilibrium policies are \((\tau_g^*(r), \phi_g^*(r))\). In this range, as \( r \) increases both public production and the tax rate increase. If (18) is not satisfied, the optimal policies are \((\tau_g^+(r), \phi_g^+(r))\). As \( r \) increases, both public production and the tax rate decrease.

- If \( r > r_g^{**} \), the solution involves unemployment. The equilibrium policies are \((\tau_g^*(r), \phi_g^*(r))\). In this range, as \( r \) increases public production decreases, the tax rate increases, and unemployment increases.

\(^{15}\) The fact that \( r \) is multiplied by \( q \) in the objective function has no effect on the optimal policies, since \( qr \) is just a constant.
4.1.2 High $q$

When $q$ exceeds $q^*_0$ and the revenue requirement is so low that the budget constraint does not bind, we are in the case represented in Fig ??B. Here, the optimal policies for the mwc lie in the feasible set below the resource constraint (see Fig ??B). Intuitively, when $q$ is large, the mwc want to keep taxes high and public production low to extract revenue to finance transfers to their districts. There is nothing to stop taxes being sufficiently high and public production sufficiently low, that unemployment arises. Optimal extractive policies in this case are:

$$(\tau^*_0, g^*_0) = \left( \frac{(q-1)A_0 - g_0}{(A_0 - \omega)(2q-1)}, \frac{\gamma}{(q-1)\omega} \right).$$

(28)

As before, define $r^*_0$ to be the revenue requirement equal to $R_0(\tau^*_0, \omega) - \omega g^*_0$. Then, if the revenue requirement $r$ is less than or equal to $r^*_0$ the mwc will choose (28). As in the low $q$ case, policies in this case are driven by the desire to extract revenue rather than to mitigate unemployment. We therefore refer to this case as unemployment with optimal revenue extraction.

If $r$ exceeds $r^*_0$ the mwc will not make transfers and the optimal policies will be given by $(\tau^*_0(r), g^*_0(r))$. Since there is unemployment at $(\tau^*_0, g^*_0)$, it must be the case that $r^*_0$ exceeds $r^*_0$, and so we will have unemployment. To summarize:

**Proposition 5** When $q > q^*_0$ the solution to problem (24) has the following properties.

- If $r \leq r^*_0$, the solution involves unemployment with optimal revenue extraction. The equilibrium policies are $(\tau^*_0, g^*_0)$ and are independent of the revenue requirement.
- If $r > r^*_0$, the solution involves unemployment. The equilibrium policies are $(\tau^*_0(r), g^*_0(r))$.
  
  In this range, as $r$ increases public production decreases, the tax rate increases, and unemployment increases.

4.2 Dynamics

As for the benevolent government case, define $r_0(b) = (1 + \rho)b - b_0(b)$ to be the revenue requirement implied by the equilibrium policies in state $\theta$ with initial debt level $b$. Letting $(\tau^*_0(r_0(b)), g^*_0(r_0(b)))$ denote the static equilibrium policies described in Propositions 4 and 5, it is clear that $(\tau_0(b), g_0(b))$ will equal $(\tau^*_0(r_0(b)), g^*_0(r_0(b)))$. Similarly to the previous section, therefore, the key issue is to identify which of the cases described in Propositions 4 and 5 will arise in the long run. This requires understanding the long run behavior of debt.
Given the equilibrium policy functions, for any initial debt level \( b \), we can define \( H(b, b') \) to be the probability that next period's debt level will be less than \( b' \). Given a distribution \( \psi_{t-1}(b) \) of debt at time \( t-1 \), the distribution at time \( t \) is \( \psi_t(b') = \int_b \psi_{t-1}(b) dH(b, b') \). A distribution \( \psi^*(b') \) is said to be an invariant distribution if

\[
\psi^*(b') = \int_b H(b, b') d\psi^*(b).
\]

If it exists, the invariant distribution describes the steady state of the government's debt distribution. We now have:

**Proposition 6** There exists a debt level \( b^* \in (r_H^*, \rho, b) \) such that the equilibrium debt distribution converges to a unique, non-degenerate, invariant distribution with full support on \([b^*, b]\). The dynamic pattern of debt is counter-cyclical: the government expands debt when private sector costs are high and contracts debt when costs are low until it reaches the floor level \( b^* \).

The floor debt level \( b^* \) described in Proposition 6 prevents the government from accumulating a sufficiently large buffer stock of assets that they do not need to issue new debt. Once the debt level has reached \( b^* \), the mwc prefers to divert surplus revenues to transfers rather than to paying down more debt. This is analogous to the results of Battaglini and Coate (2008, 2010) for the tax smoothing model. The debt level \( b^* \) depends on the fundamentals of the economy and can be characterized following the analysis in Battaglini and Coate, but these details are not central to our mission here and so we relegate them to the Appendix. For now, we will simply assume that it is positive, which seems empirically relevant case.

In terms of revenue requirements, Proposition 6 implies that in steady state \( r_H(b) \) is less than \( \rho b \) and \( r_L(b) \) is at least as big as \( \rho b \) with the inequality holding strictly for \( b \) larger than \( b^* \).

Two further important properties are established in the Appendix. First, for each state \( \theta \), \( r_\theta(b) \) is increasing in \( b \), so that higher debt levels result in greater fiscal pressure on the government. Second, the floor debt level \( b^* \) is such that \( r_H(b^*) \) strictly exceeds \( r_H^* \) while \( r_L(b^*) \) is less than or equal to \( r_L^* \). This implies that the economy never reaches the first cases of Propositions 4 and 5 in the high cost state, but will do so in the low cost state for sufficiently low debt levels. Letting \( b_L^* \) be the debt level such that \( r_L(b_L^*) \) is equal to \( r_L^* \), this set of "sufficiently low debt levels" is \([b^*, b_L^*]\).

Turning to employment levels, from Proposition 6, we know that when \( q \) exceeds \( q_\theta^* \), we will necessarily have unemployment in state \( \theta \) whatever revenue requirement the government faces.
From Proposition 5, when \( q \) is less than \( q^*_L \), we will have unemployment in state \( \theta \) whenever the revenue requirement exceeds \( r^*_L \). This will occur with positive probability if \( r^*_L(b) \) exceeds \( r^*_L \). Since \( r^*_L(b) \) is equal to \( \max_r R_H(r, \omega) \), a sufficient condition for unemployment to arise in the high cost state is that private sector labor demand would be less than the number of workers if taxation is at the peak of the Laffer curve. Formally, this amounts to the condition that \( n \omega \) exceeds \( n \omega (A_H - \omega) / 2 \xi \) which is implied by Assumption 1. Assumption 1 does not guarantee that \( r^*_L(b) \) exceeds \( r^*_L \), and so it is possible that unemployment does not occur in the low cost state when \( q \) is less than \( q^*_L \). For our next result, it is convenient to define \( b^*_L \) to be the maximal \( b \in [b^*, b] \) such that \( r^*_L(b) \) is lower or equal than \( r^*_L \). In the high cost state, \( b^*_H \) is less than \( b^*_L \), while in the low cost state it could be that \( b^*_L \) equals \( b^* \).

Combining these observations concerning revenue requirements with Propositions 4 and 5, yields the following result:

**Proposition 7** The equilibrium debt distribution has the following implications for employment levels and policies.

- If \( q > q^*_L \), there is unemployment in the high cost state. In the low cost state, there is unemployment with optimal revenue extraction if \( b \in [b^*, b^*_L] \) and unemployment if \( b > b^*_L \). Unemployment is increasing in \( b \) in the high cost state and in the low cost state when \( b > b^*_L \). For given \( b \), unemployment is greater in the high than the low cost state.

- If \( q \in (q^*_H, q^*_L) \), there is unemployment in the high cost state. In the low cost state, there is full employment with optimal revenue extraction if \( b \in [b^*, b^*_L] \), full employment with distortions if \( [b^*_L, b^*_L^*] \), and unemployment if \( b > b^*_L^* \). Unemployment is increasing in \( b \) in the high cost state and in the low cost state when \( b > b^*_L^* \). For given \( b \), unemployment is greater in the high than the low cost state.

- If \( q \) is less than \( q^*_H \), in the high cost state, there is full employment with distortions if \( [b^*, b^*_H] \) and unemployment if \( b > b^*_H^* \). In the low cost state, there is full employment with optimal revenue extraction if \( b \in [b^*, b^*_L] \), full employment with distortions if \( [b^*_L, b^*_L^*] \), and unemployment if \( b > b^*_L^* \). Unemployment is increasing in \( b \) in the high cost state when \( b > b^*_H^* \) and in the low cost state when \( b > b^*_L^* \). For given \( b \), unemployment is greater in the high than the low cost state when \( b > b^*_H^* \).
Proposition 7 shows that the model is consistent with a variety of possible employment patterns ranging from unemployment in both high and low cost states to full employment in both states except at high debt levels. There are two general lessons from the Proposition. First, for any given debt level in the support of the equilibrium distribution, employment levels are weakly higher in the low cost state and strictly higher for debt levels above some critical level. What this means is that negative shocks to the private sector translate into higher unemployment levels. Second, employment levels in both states are weakly decreasing in the economy’s debt level and strictly so for debt levels above a critical level. Thus, higher debt leads to greater unemployment.

To illustrate the workings of the model, consider the case in which q is between q_L and q_H. In this case, there is always unemployment in high cost states but full employment in low cost states for suitably low debt levels. In this case, the government mitigates unemployment in high cost states by issuing debt. If b is less than b_L, then a return to the low cost state will be sufficient to restore full employment. If the economy is in the high cost state for a sufficiently long period of time, however, debt will increase beyond this level and unemployment will persist even when the low cost state returns. Eventually, however, the economy returns to full employment. When the low cost state returns, the legislators reduce debt. If the low cost state persists, debt will fall below b_L. At this point, full employment is achieved with activist fiscal policies. When debt falls below b_L, we achieve the optimal policies for the mwc and transfers will be made. Job creation no longer drives policies.

4.3 Equilibrium stimulus plans

In the steady state of the political equilibrium, when private sector costs are high, the government expands debt and the funds are used to mitigate unemployment.\(^{16}\) The government therefore employs fiscal stimulus plans, as conventionally defined. By studying the size of these stimulus plans and the changes in policy they finance, we can obtain a positive theory of fiscal stimulus. More specifically, in a recession, we can interpret ρb − τ_H(b) as the magnitude of the stimulus, since this measures the amount of additional resources obtained by the government to finance fiscal policy changes. It is of interest to understand how this depends on the initial debt level b. An understanding of how the stimulus funds are used can be obtained by comparing the equilibrium

\(^{16}\) Even when q is less than q_H and b is less than b_L, there will be unemployment prior to government stimulus if b_H is positive. This follows from Assumption 1.
tax and public good policies with the policies that would be optimal if the debt level were held constant.

The simplest case to consider is when the stimulus package does not completely eliminate unemployment. This must be the case when \( q > q_H^* \). Moreover, even when this is not the case, unemployment will remain post-stimulus whenever the initial debt level exceeds \( b_H^* \). Drawing on the analysis in Section 3, Fig 4 illustrates what happens in this case. From Propositions 4 and 5, the policies that would be chosen if the debt level were held constant are \((\tau_H(\rho b), \tilde{g}_H(\rho b))\). The reduction in the revenue requirement made possible by the stimulus funds, shifts the budget line up and permits a new policy choice \((\tau_H(r_H(b)), \tilde{g}_H(r_H(b)))\). As discussed in Section 3, in the unemployment range, the tax rate is increasing in the revenue requirement and public production is decreasing. Thus, we know that \( \tau_H(r_H(b)) \) is less than \( \tau_H(\rho b) \) and that \( \tilde{g}_H(r_H(b)) \) exceeds \( \tilde{g}_H(\rho b) \), implying that stimulus funds will be used for both tax cuts and increases in public production.

In terms of effectiveness, we know from Section 3 that if \( \tau_H(r_H(b)) \) is less than \( \tau_H^* \), then reducing the tax cut slightly and using the revenues to finance a slightly larger public production increase will produce a bigger reduction in unemployment. Conversely, if \( \tau_H(r_H(b)) \) exceeds \( \tau_H^* \) then reducing the public production increase and using the revenues to finance a slightly larger tax cut will produce a bigger reduction in unemployment. That the government chooses a tax rate
different than \( \tau_H^* \) reflects the fact that it cares not only about unemployment but also the mix of public and private outputs. When \( \bar{\tau}_H(\bar{r}_H(b)) \) is less than \( \tau_H^* \) it prefers a smaller level of public production than that associated with the tax rate \( \tau_H^* \), while if \( \bar{\tau}_H(\bar{r}_H(b)) \) exceeds \( \tau_H^* \) it prefers a smaller level of public production than that associated with the tax rate \( \tau_H^* \). Both situations are possible depending on the parameters. Whether \( \bar{\tau}_H(\bar{r}_H(b)) \) is less than or greater than \( \tau_H^* \) will depend partially on the initial debt level. As \( b \) approaches \( \bar{b} \), the government will always choose too small a tax cut. This is irrespective of its preferences as measured by gamma. This reflects that as \( b \) approaches \( \bar{b} \), the equilibrium tax rate approaches \( 1/2 \) which exceeds \( \tau_H^* \).

One way of thinking about these results concerning the comparison of \( \bar{\tau}_H(\bar{r}_H(b)) \) and \( \tau_H^* \) is in terms of multipliers. It is commonplace in the empirical literature to try evaluate the multipliers associated with different stimulus measures. The multiplier associated with a particular stimulus measure is defined to be the change in GDP divided by the budgetary cost of the measure. In our model, GDP equals private sector output plus the cost of public production. When there is unemployment, the public production multiplier is 1 and the tax cut multiplier is approximately \( A_H / (1 - 2\bar{\tau}_H(\bar{r}_H(b))) (A_H - \bar{w}) \).\(^{17}\) This will exceed the public production multiplier if \( \bar{\tau}_H(\bar{r}_H(b)) \) exceeds \( \tau_H^* \) and be less than the public production multiplier if \( \bar{\tau}_H(\bar{r}_H(b)) \) is less than \( \tau_H^* \). The analysis illustrates why we should not expect the government to choose policies in such a way as to equate multipliers across instruments. Tax cuts and public production increases have different implications for the mix of public and private outputs. A further point to note is that the tax multiplier is highly non-linear. Tax cuts will be more effective the larger is the tax rate and the tax rate reflects the economy's initial debt level.

When the stimulus package eliminates unemployment as would be the case when \( q^* \) exceeds \( q_H^* \) and the initial debt level is less than \( b_H^* \), matters are more complicated. This is because of the non-monotonic behavior of policies in the full employment with distortions range identified in Proposition 1. In particular, we will not necessarily get both tax cuts and an increase in public production. Fig 6 illustrates a case in which the stimulus package involves not only using all the stimulus funds to increase public production but also increasing taxes to supplement the stimulus funds. Fig 7 illustrates a case in which the stimulus package involves decreases in both taxes and public production. The model is therefore consistent with a variety of possible stimulus plans.

\(^{17}\) Give derivations and also explain what happens when there is not unemployment.
5 Conclusion

This paper has explored the interaction between fiscal policy and unemployment. It has developed a dynamic economic model in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing debt. In the context of this model, the paper has analyzed the simultaneous determination of fiscal policy and unemployment in long run equilibrium. Outcomes with both a benevolent government and political decision-making are considered.

With a benevolent government, there would be no unemployment in the long run and the mix of public and private outputs would be optimal. The way in which a benevolent government achieves this is by accumulating bond holdings and using the earnings from these assets to finance unemployment mitigation when the private sector experiences negative shocks. The benevolent government solution is provocative in suggesting it is not enough to assume market imperfections to get a theory of unemployment. One also needs to explain why government does not use fiscal policy to circumvent these imperfections.

With political decision-making, the model delivers an appealing positive theory of fiscal policy and unemployment. The theory predicts that the government will expand debt when the private sector experiences negative shocks and use the funds to mitigate unemployment. In normal times, the government will contract debt until it reaches a floor level. The existence of this floor level reflects the fact that politicians prefer to divert surplus revenues to transfers rather than to paying down more debt. Depending on the extent of political frictions, the theory predicts there can never be full employment, full employment in normal times for low debt levels, or even full employment when there are negative shocks for sufficiently low debt levels. In all cases, employment levels are weakly decreasing in the economy’s debt level and strictly so for debt levels above a critical level. Thus, the theory predicts that higher debt leads to higher unemployment.

The theory not only tells us that the government will introduce stimulus plans in recessions, it also sheds light on the uses of these funds and their effectiveness in reducing unemployment. While stimulus funds will generally be used to finance both tax cuts and public production increases, the model allows for the possibility that stimulus will be accompanied by either tax hikes or cuts in public production. An important insight from the theory is that in general the mix of tax cuts and public production increases will not achieve the maximum possible reduction in unemployment.
The government will balance the goals of minimizing unemployment and achieving the ideal mix of public and private outputs. This may involve a tax cuts higher or lower than the unemployment minimizing tax cuts depending upon society's preferences for the public good. In terms of the multipliers that are the focus of the empirical literature, the theory suggests that we should not expect to see parity between the tax cut and public production multipliers. Moreover, the theory suggests that the relationship between the size of these multipliers is ambiguous and will depend upon the debt position of the economy.