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Entanglement, decoherence, and dynamics of a two-state system

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Entanglement and decoherence arguably define the central issues of concern in present day quantum information theory. Decoherence occurs when a system interacts with its environment in an irreversible way; this prevents the quantum superposition of the system + environment's wavefunction from interfering with each other. A better understanding of environment-induced destruction of coherent superposition states is needed, as well as a clear description of the degree of entanglement between the quantum system and its environment. We quantitatively establish a correspondence between entanglement, decoherence, and spin dynamics for a two-state system coupled to a bath of harmonic oscillators, resulting in the celebrated spin-boson model. Applications to solid-state and cold atomic systems are also discussed.

Keywords: entanglement; decoherence; spin-boson model; spin dynamics

1. Introduction

The modern view of quantum mechanics is that there is no fully isolated quantum systems [1,2]. When we measure one, we implicitly integrate out its environment. Here, we want to devote the talk to the following questions: what is the relationship between lack of isolation and entanglement, and what is the connection between the degree of entanglement and the destruction of the quantum superposition states?

Those questions can be addressed from the prototype spin-boson model, where a two-level system is subject to a bath of harmonic oscillators [3,4]:

$$H = -\frac{\hbar\Delta}{2}S_x - \frac{\epsilon}{2}S_z + \frac{q_0}{2}S_z\sum_i c_i x_i + H_{\text{osc}} \qquad (1)$$

with

$$H_{\text{osc}} = \sum_i\left(\frac{p_i^2}{2m_i} + \frac{1}{2}m_i\omega_i^2 x_i^2\right). \qquad (2)$$

Dissipation in quantum mechanics represents an important statistical mechanical problem having ramifications in systems as diverse as biological systems to limitations of quantum computation [5,6]. Here, Δ is the (bare) tunneling matrix element, ϵ the bias (detuning), x_i, p_i, m_i, and ω_i are the coordinate, momentum, mass, and frequency of the ith harmonic oscillator. Additionally, S_j where $j = x, y, z$ are Pauli matrices so that the eigenstate of S_z with eigenvalue +1 (−1) corresponds to the system being in the $|\uparrow\rangle$ ($|\downarrow\rangle$) state or being localized in the right (left) well in a double-well analogy (see Figure 1). We closely follow the notations of [3]. Here, c_i represents the strength of the coupling with the ith oscillator and q_0 is a (constant) parameter which can be set to one [3]. Complete information about the effect of the environment is encapsulated in the spectral function [3]: $J(\omega) = (\pi/2)\sum_i (c_i^2/m_i\omega_i)\delta(\omega - \omega_i)$. Below, we consider the case of Ohmic dissipation where $J(\omega) = \eta\omega\exp(-\omega/\omega_c)$ and ω_c represents a high-frequency cutoff. The quantity η is the dissipation (friction) coefficient. In fact, it is convenient to build a dimensionless measure of the system–environment coupling, $\alpha = \eta q_0^2/2\pi\hbar$ [3]. We focus on the delocalized regime $(0 < \alpha < 1)$ allowing a quantum spin superposition. The point $\alpha^* = 1/2$ corresponds to the exactly solvable Toulouse limit [3,4].

An environment generally induces some uncertainty in the spin direction which is responsible for quantum decoherence, i.e. for the rapid vanishing of the off-diagonal elements of the spin reduced density matrix. On the other hand, a finite coupling between the spin and its environment also induces entanglement. Combining exact calculations [7,8] and simple physical arguments, we quantify the relationship between entanglement and decoherence in the spin-boson model. We study the case close to zero detuning $\epsilon/\Delta \approx 0$ such that the quantum superposition is ideal.

We show that for $\alpha^* \leq \alpha(<1)$, the spin and the bath become (almost) *maximally* entangled and the off-diagonal elements of the ground state spin reduced density matrix (which can be accessed via persistent

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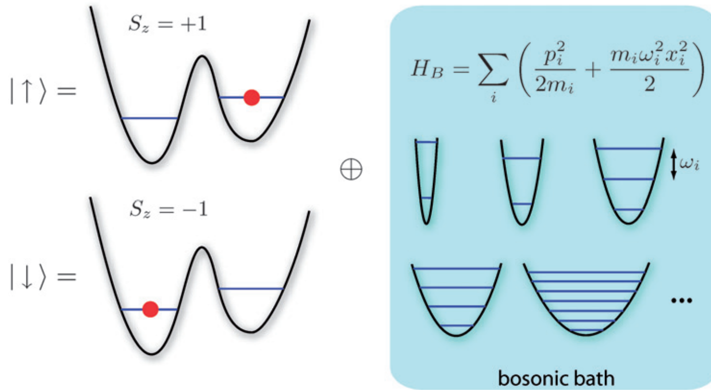

Figure 1. Double-well formulation of the two-level system subject to a bath of oscillators. Here, the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of the Pauli operator S_z which take the eigenvalues ± 1. (The color version of this figure is included in the online version of the journal.)

current measurements [9]) $\sim\mathcal{O}(\Delta/\omega_c)$ become as small as in the localized regime $\alpha > 1$ where the quantum spin superposition is (almost) lost [7–9]. Decoherence may also be defined as the environment-induced suppression of the spin oscillations produced by the spin Larmor precession when applying a perpendicular magnetic field. In this context, $\alpha^* = 1/2$ also refers to the coherent-incoherent crossover where Rabi oscillations of the spin vanish [3,4]. We present new results [10] extending the Non-Interacting Blip Approximation (NIBA) of [3]. Those results emphasize the connection between maximal entanglement and quantum decoherence in the spin-boson model.

2. General considerations

Assuming that the two-state system would be isolated from the large environment ($\alpha = 0$) the quantum mechanical problem is easily solvable. In particular, the eigenstates take the precise form:

$$\begin{aligned} |g\rangle &= \cos\tfrac{\theta}{2}|\uparrow\rangle + \sin\tfrac{\theta}{2}|\downarrow\rangle, \\ |e\rangle &= -\sin\tfrac{\theta}{2}|\uparrow\rangle + \cos\tfrac{\theta}{2}|\downarrow\rangle, \end{aligned} \tag{3}$$

where the angle θ obeys $\theta = \arctan(\hbar\Delta/\epsilon)$ and the corresponding eigenenergies are $\pm E$ where $E = \frac{1}{2}[\epsilon^2 + (\hbar\Delta)^2]^{1/2} > 0$. The eigenenergy $-E$ corresponds to the energy of the ground state $|g\rangle$ whereas $+E$ corresponds to the energy of the excited state $|e\rangle$. Below, we are mainly interested in the limit $\epsilon/\Delta \to 0$ or $\theta \to \pi/2$ at zero temperature. This results in the well-known even- and odd-parity combinations:

$$\begin{aligned} |g\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \\ |e\rangle &= \frac{1}{\sqrt{2}}(-|\uparrow\rangle + |\downarrow\rangle). \end{aligned} \tag{4}$$

If one prepares the system in the $|\uparrow\rangle$ state at time $t = 0$ then one finds:

$$\langle S_z\rangle_t = \cos(\Delta t). \tag{5}$$

This oscillation effect has no classical analog: this is a consequence of the phase coherence between the amplitudes for being in the left or right well. By switching on the coupling with the bath ($\alpha \neq 0$), the decoherence at $\alpha^* = 1/2$ will appear in two ways: a suppression of these oscillations (Section 4) and a reduction of the off-diagonal elements of the ground state spin reduced density matrix (Section 3).

Now, we need to define quantitatively the entanglement between the qubit and its environment. At zero temperature, the 'universe' (=qubit + environment) is in a pure state and then a useful measure of entanglement is the von Neumann entanglement entropy, involving the spin reduced density matrix ρ[11,12]:

$$S = -\mathrm{Tr}(\rho \log_2 \rho). \tag{6}$$

We have chosen the $\log_2$ basis such that the maximal entropy corresponds to $S_{\max} = 1$ ($S_{\max} = 1$ corresponds to the entanglement entropy of the Einstein–Podolsky–Rosen pair). The spin reduced density matrix takes the general form,

$$\rho = \mathrm{Tr}_{\mathrm{Bath}}|\Psi_{\mathrm{G}}\rangle\langle\Psi_{\mathrm{G}}|, \tag{7}$$

where the ground state wavefunction reads (for $\epsilon \approx 0$, $\rho_{\uparrow\uparrow} = \rho_{\downarrow\downarrow} = 1/2$) [3]:

$$|\Psi_{\mathrm{G}}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\chi_\uparrow(\alpha)\rangle + |\downarrow\rangle|\chi_\downarrow(\alpha)\rangle\big). \tag{8}$$

Here, $|\chi_{\uparrow,\downarrow}(\alpha)\rangle$ are the states of the environment. It is also useful to introduce the eigenvalues p_+ and p_- of the spin reduced density matrix such that [7,8]:

$$p_\pm = \left(1 \pm \left[\langle S_x\rangle^2 + \langle S_z\rangle^2\right]^{1/2}\right)\Big/2. \tag{9}$$

We have defined $\langle S_i\rangle = \langle\Psi_G|S_i|\Psi_G\rangle$ and we have taken into account that $\langle S_y\rangle = 0$ since the spin-boson Hamiltonian is invariant under the transformation $S_y \to -S_y$. The degree of entanglement then takes the form:

$$S = -p_+ \log_2 p_+ - p_- \log_2 p_-. \quad (10)$$

Here, p_+ (p_-) can be interpreted as the probability that the two-level system is in its ground (excited) state when coupling to the bosonic environment. At small α, we observe that p_- becomes non-zero and it follows $\alpha \ln(\omega_c/\Delta)$ [13].

3. Entanglement and spin observables for $\epsilon \approx 0$

When $\alpha = 0$, the qubit is decoupled from the environment implying that $|\Psi_G\rangle$ is a product state, i.e. $|\chi_\uparrow(0)\rangle = |\chi_\downarrow(0)\rangle$ and we can easily check that there is no entropy. The spin lies in a pure state, i.e. on the surface of the Bloch sphere $\langle S_x\rangle^2 + \langle S_z\rangle^2 = 1$ implying that $p_+ = 1$ and $p_- = 0$, and therefore $S(\alpha = 0) = 0$.

Now, let us consider $\alpha \neq 0$ and the limit $\epsilon/\Delta \to 0$ such that $\langle S_z\rangle \to 0$ (for $\alpha < 1$ [3]). The degree of entanglement defined in Equation (10) can be found by computing $\langle\Psi_G|S_x|\Psi_G\rangle$. Using the ground state wavefunction of Equation (8), we identify:

$$\langle S_x\rangle = \tfrac{1}{2}\big(\langle\chi_\uparrow(\alpha)|\chi_\downarrow(\alpha)\rangle + \langle\chi_\downarrow(\alpha)|\chi_\uparrow(\alpha)\rangle\big) = \langle\chi_\uparrow(\alpha)|\chi_\downarrow(\alpha)\rangle. \quad (11)$$

To compute $\langle S_x\rangle$, first one can try the adiabatic approximation [3,4]. For $\Delta = 0$, the zeroth-order adiabatic approximation, one easily finds [3,4]:

$$|\chi_{\uparrow,\downarrow}\rangle(\Delta = 0) = \prod_i \exp\left(\pm\frac{\mathrm{i}}{2}\Omega_i\right)|O_i\rangle \quad \text{with}$$
$$\Omega_i = \frac{q_0 c_i}{\hbar m_i \omega_i^2} p_i, \quad (12)$$

where $|O_i\rangle$ is the ground state of the corresponding oscillator. For finite Δ, this approximation is justified if one includes only the oscillators with a frequency $\omega_i > \Delta$. Then, one gets the exponential dressing Franck–Condon factor F [14]:

$$\langle S_x\rangle = F = \exp - \int_{\omega_{\min}\sim\Delta}^{+\infty} \frac{J(\omega)}{\omega^2}\,\mathrm{d}\omega = \exp\left[-\alpha \ln\left(\frac{\omega_\mathrm{c}}{\omega_{\min}}\right)\right]. \quad (13)$$

On the other hand, this formula completely disregards slow modes (that are slow on the scale Δ^{-1}). To improve the procedure, one can apply the standard adiabatic 'renormalization' scheme to lower frequencies [3,4]. For this purpose, it is convenient to observe that the dressed tunneling matrix elements take the form $\Delta_r = \Delta\langle\chi_\uparrow(\alpha)|\chi_\downarrow(\alpha)\rangle$. This result is consistent with Equation (11) and it can also be obtained through the polaronic transformation $U = \exp(-\mathrm{i}S_z\Omega/2)$ and $\Omega = \sum_i \Omega_i$ [4]. In the transformed Hamiltonian $H' = UHU^{-1}$, the effect of the bath is only included through the modification $S_+ \to S_+\exp(-\mathrm{i}\Omega)$ and $S_- \to S_-\exp(\mathrm{i}\Omega)$. To compute ground state properties, then this is equivalent to replace Δ by Δ_r [3,4]. Following the adiabatic renormalization scheme, we find $\langle S_x\rangle \approx \Delta_r/\Delta$ where $\Delta_r = \Delta(\Delta/\omega_c)^{\alpha/1-\alpha}$. We have taken into account modes of the environment with excitation frequencies $\sim\Delta_r$. This scheme should give a reasonable result at small $\alpha \ll 1$. For $\alpha = 1$, the spin-boson model displays a dissipative transition to a localized phase where $\langle S_z\rangle \sim 1$ at arbitrarily small $\epsilon > 0$ [3]. Since $\langle S_x\rangle$ is continuous at the transition, the (small) term $\langle S_x\rangle = \Delta/((2\alpha-1)\omega_c)$ [7] from the localized phase may spread out in the delocalized region. Thus, for $0 \le \alpha < 1$, one expects:

$$\langle S_x\rangle = \langle\chi_\uparrow(\alpha)|\chi_\downarrow(\alpha)\rangle = \frac{1}{2\alpha - 1}\frac{\Delta}{\omega_\mathrm{c}} + C(\alpha)\frac{\Delta_\mathrm{r}}{\Delta}. \quad (14)$$

The second part stemming from the adiabatic renormalization should be important at small α whereas the first part is a reminiscence of the localized phase.

Now, to compute the coefficient $C(\alpha)$ and prove Equation (14) we use an exact mapping between the spin-boson model and the interacting resonant level model [9]. The ground state energy E_G of the system then can be obtained exactly through the Bethe Ansatz method [15,16]. Using that $\langle S_x\rangle = -(2/\hbar)\partial E_G/\partial\Delta$ we find [7–9]:

$$C(\alpha) = \frac{\exp[-b/(2-2\alpha)]}{\pi^{1/2}(1-\alpha)} \frac{\Gamma[1 - 1/(2-2\alpha)]}{\Gamma[1 - \alpha/(2-2\alpha)]} \left(\frac{\omega_\mathrm{c}}{D}\right)^{\alpha/1-\alpha}, \quad (15)$$

Γ refers to the Γ function and we have defined:

$$b = \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha), \quad (16)$$

and the relationship between ω_c and the parameter D is given, e.g. in [9]. For $\alpha \to 0$, we find $C(\alpha) \to 1$ which is consistent with the adiabatic renormalization scheme. This implies that the entanglement (entropy) grows *linearly* with α (with a non-universal slope proportional to $\ln(\omega_c/\Delta)$) and the quantum coherence of the system is preserved since the off-diagonal elements of the spin reduced density matrix satisfy $\langle S_x\rangle = \langle\exp(\mathrm{i}\Omega)\rangle = 1 - \mathcal{O}(p_-)$ [13]; see Equation (9). As we turn on the coupling with the environment we introduce some uncertainty in the spin direction, and $\langle S_x\rangle$ progressively decreases. For small $\alpha \ll 1/2$ the quantum superposition of the system is relatively well preserved and the qubit behaves almost as in the ideal isolated case. However, the decoherence becomes drastically suppressed when $\alpha = 1/2$. Since $(4/\pi)\Gamma(1-2\alpha) \to 4/(\pi(1-2\alpha))$ and $D(\alpha = 1/2) = 4\omega_c/\pi$, the two terms in Equation (14) almost cancel each other in agreement with the results from the non-interacting resonant level model [7–9]. For $\alpha > 1/2$, the first (small) term in Equation (14) (reminiscent of the localized phase) dominates and the quantum decoherence of the spin becomes almost complete $\langle S_x \rangle = \langle \exp(i\Omega) \rangle \sim \Delta/\omega_c \approx 0$ [7–9].

For $\alpha^* \leq \alpha < 1$ where $\alpha^* = 1/2$, the 'universe' (=qubit+environment) behaves as a quantum incoherent system. The persistent current (or$\langle S_x \rangle$) becomes as tiny as in the localized phase where the quantum superposition is (almost) lost. Assuming $\epsilon/\Delta_r \to 0$ ($\langle S_z \rangle = 0$), this happens when the entanglement entropy is maximized: $\langle S_x \rangle \approx \mathcal{O}(\Delta/\omega_c)$ leads to the maximal uncertainty $p_\pm \sim 1/2$ and as a result $S \to 1$ [7,8]. It is useful to observe the connection between maximal entropy and irreversibility or destruction of quantum superposition states. The uncertainty (in Ω or in the spin state) produces decoherence (dephasing) and entropy. Those results have been confirmed by the Numerical Renormalization Group (NRG) [7,17].

4. Spin dynamics and coherent-incoherent crossover

Another useful quantity in the context of macroscopic quantum coherence is the occupation probability $P(t) = \langle S_z(t) \rangle$ (here, we assume the Heisenberg representation), where the system is subject to the non-equilibrium initial preparation $S_z(t=0) = +1$ and the initial density matrix is in a factorized form [3,4]. This preparation of the initial state may be realized by the application of a large external bias $-\epsilon_0\theta(-t)$ with $\epsilon_0 \gg \Delta$. At time $t = 0$, the dynamics starts out. Assuming a factorized initial state, the preparation of the bath is not so important; the canonical equilibrium of the bare Hamiltonian with $\alpha = 0$ and the shifted thermal equilibrium distribution give the same result for time scales larger than $1/\omega_c$ [4].

At $\alpha^* = 1/2$, the spin oscillations (defined in Equation (5)) will disappear. This result is obtained from a weak-coupling expansion (one-phonon process) [18] or from the Non-Interacting Blip Approximation (NIBA) [3,4]. But, those approaches neglect the spin's back-action onto the bath. Below, we show that a rigorous result can be obtained rephrasing the problem as that of a (non-)unitary time evolution of a quantum state vector under the influence of a random Gaussian perturbation [10].

Following [1], the influence functional takes the precise form [3]:

$$F[\xi, \chi] = \exp\left(-\frac{q_0^2}{\pi\hbar} \int_{t_0}^{t} d\tau \int_{t_0}^{\tau} ds[-iL_1(\tau - s)\xi(\tau)\chi(s) + L_2(\tau - s)\xi(\tau)\xi(s)] \right). \tag{17}$$

The functions L_1 and L_2 are related to the spectral function $J(\omega)$ [3]. Let us introduce the states $A = (\uparrow, \uparrow)$ and $D = (\downarrow, \downarrow)$ corresponding to the diagonal elements of the spin reduced density matrix, as well as the off-diagonal elements $B = (\uparrow, \downarrow)$ and $C = (\downarrow, \uparrow)$. The double path integral can be viewed as a single path that visits the four states. A period the path dwells in a diagonal (off-diagonal) state is called a *sojourn* (*blip*). During a sojourn the function $\xi(\tau)$ is zero and during a blip $\chi(\tau) = 0$. The sojourn states are labelled $\chi(\tau) = \pm 1$ for A and D, respectively, and the blip states obey $\xi(\tau) = \pm 1$ for B and C, respectively. For trajectories with $2n$ 'flips' (transitions) the function $\xi(\tau)$ becomes $\xi(\tau) = \sum_{j=1}^{2n} \Xi_j \theta(\tau - t_j)$ and $\chi(\tau) = \sum_{j=0}^{2n} \Upsilon_j \theta(\tau - t_j)$, where Ξ_j, $\Upsilon_j = \pm 1$. Performing the time integration over τ and s then gives a quadratic form in the variables Ξ, Υ [3]. In the NIBA, the time spent in a diagonal state is assumed to be much longer than the time in an off-diagonal state (the interblip correlations $\Xi_j \Xi_k$ are neglected) and the probability $P(t)$ to be in the state A ($|\uparrow\rangle$) is found via the Laplace transform [3]:

$$P(t) = \frac{1}{2\pi i} \int_C d\lambda \exp(\lambda t) \frac{1}{\lambda + f(\lambda)}. \tag{18}$$

Here C is the standard Bromwich contour and in the scaling limit $\Delta/\omega_c \ll 1$ [3]:

$$f(\lambda) = \Delta_e \left(\frac{\Delta_e}{\lambda} \right)^{1-2\alpha}, \tag{19}$$

where $\Delta_e = \Delta_r (\cos(\pi\alpha)\Gamma(1-2\alpha))^{1/2(1-\alpha)}$.

For $\alpha \to 0$, one recovers the persistent oscillations $P(t) = \cos(\Delta t)$ whereas for $\alpha = 1/2$ one gets a pure relaxation $P(t) = \exp-(\pi\Delta^2\, t/(2\omega_c))$, which is in perfect accordance with the non-interacting resonant level model [3,4]. For $0 < \alpha < 1/2$, the spin still displays coherent oscillations (due to a conjugate pair of simple poles) leading to $P_{\text{coh}}(t) = a\cos(\zeta t + \phi)\exp(-\Gamma t)$ and the quality factor obeys [3,4]:

$$\frac{\zeta}{\Gamma} = \cot\left(\frac{\pi\alpha}{2(1-\alpha)} \right). \tag{20}$$

This emphasizes that (for no detuning) $\alpha = 1/2$ corresponds to the coherent–incoherent crossover. The NIBA also predicts a purely incoherent contribution for $0 \ll \alpha \ll 1/2$ and $\Delta_e t \gg 1$, $P_{\text{inc}}(t) \propto 1/(\Delta_e t)^{2-2\alpha}$

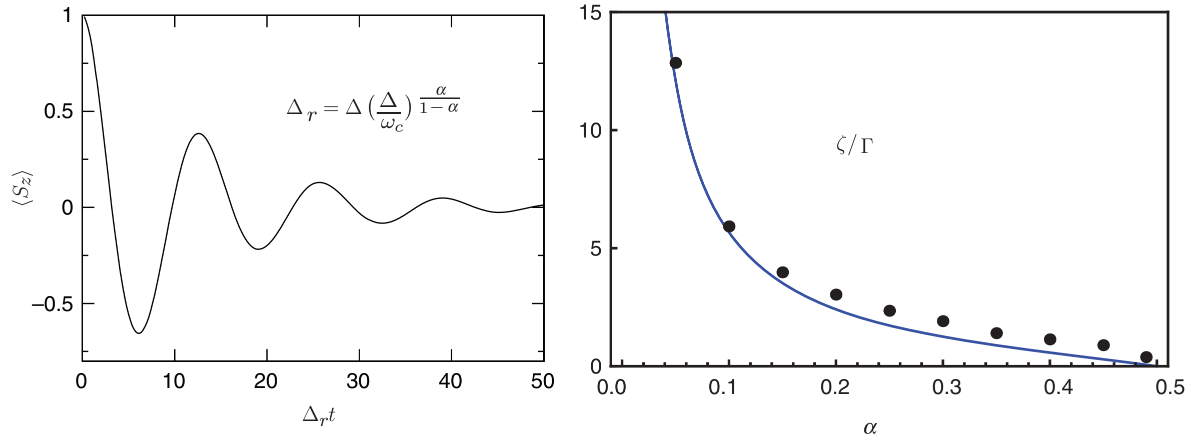

Figure 2. $\langle S_z(\Delta_r t)\rangle$ at zero temperature for $\alpha = 0.1$ (left) and ratio ζ/Γ as a function of α (right) obtained by an (exact) extension of the NIBA [10]. For $0 \le \alpha \le 1/2$, we confirm that there is a coherent damped oscillation where the ratio ζ/Γ tends to follow Equation (20). Oscillations of the spin vanish for $\alpha \to 1/2 = \alpha^*$. (The color version of this figure is included in the online version of the journal.)

stemming from the branch point at $\lambda = 0$ [3,4]. Such an incoherent term has not been found by using a conformal theory type approach [19]. The authors of [19] only report a damped oscillation similar to $P_{\mathrm{coh}}(t)$. Using an extension of [20], we have written $P(t)$ as an average over Gaussian fluctuations of a non-unitary time evolution of a four-component vector; the latter is built from the states A, B, C, D and the Gaussian fluctuations represent Hubbard–Stratonovich variables used to linearize the exponential in the functions Ξ and Υ. Our results of Figure 2 support the ratio ζ/Γ of Equation (20) for $0 \le \alpha \le 1/2$, confirming that $\alpha = 1/2$ is the incoherent crossover for the quantum (Rabi type) spin oscillations. More details on the method and further results will be shown in a forthcoming publication [10]. Finally, it should be noted that $P(t)$ is distinct from the *equilibrium* autocorrelation function of S_z [3,4].

5. Conclusion and measurement

We have shown a precise correspondence between maximal entanglement and quantum decoherence for the two-state system subject to a large collection of harmonic oscillators, at zero temperature. This happens when the dimensionless dissipation strength approaches the (Toulouse) limit $\alpha^* = 1/2$. For symmetric wells ($\epsilon/\Delta_r \approx 0$), the maximal entanglement with the bath *simultaneously* implies a prominent suppression of the off-diagonal elements of the spin reduced density matrix which become small ($\sim\mathcal{O}(\Delta/\omega_c)$) despite the delocalized nature of the ground state. The bath of harmonic oscillators in the quantum limit induces uncertainty (entropy) and decoherence. Quantum decoherence at $\alpha^* = 1/2$ can also refer to the disappearance of the quantum Rabi oscillations of the spin [3]. This shows that the entanglement with the environment changes drastically the spin dynamics from that of the isolated system. *To summarize*, $\alpha^* = 1/2$ *represents the crossover to a strongly entangled and purely incoherent regime*. For $\alpha \ll 1/2$, since the entanglement only rises linearly with α, the quantumness of the qubit is well preserved. Those results should apply for temperatures smaller than Δ_r [3,7]. The entanglement is also a useful concept to characterize the dissipative (localization) transition at $\alpha_c \approx 1$ [3]. Indeed, the entanglement entropy displays a non-universal jump which is reminiscent of the behavior of $\langle S_z \rangle$ [7,8,21]. This jump implies a Kosterlitz–Thouless type phase transition. For second-order quantum phase transitions occurring, e.g. in the one-mode superradiance (Dicke) model and in the sub-ohmic spin-boson model [22], the entanglement entropy exhibits a cusp [7].

The spin-boson model considered here can be realized for example in noisy charge qubits (see Figure 3) composed of mesoscopic quantum dots or Cooper pair boxes [5,6]. The gate voltage controls the detuning ϵ and Δ corresponds to the tunnelling amplitude between the dot and the lead(s) or the Josephson coupling energy of the junction. If the gate voltage source is placed in series with an external resistor, which can be modelled by a long LC transmission line, this may describe the spin-boson model with Ohmic dissipation [9,23]. A one-dimensional Luttinger reservoir could also be used [23,24]. Charge measurements could yield the quantity $\langle S_z \rangle$, which represents the occupation of the dot or island. In a ring geometry, the application of a magnetic flux generates

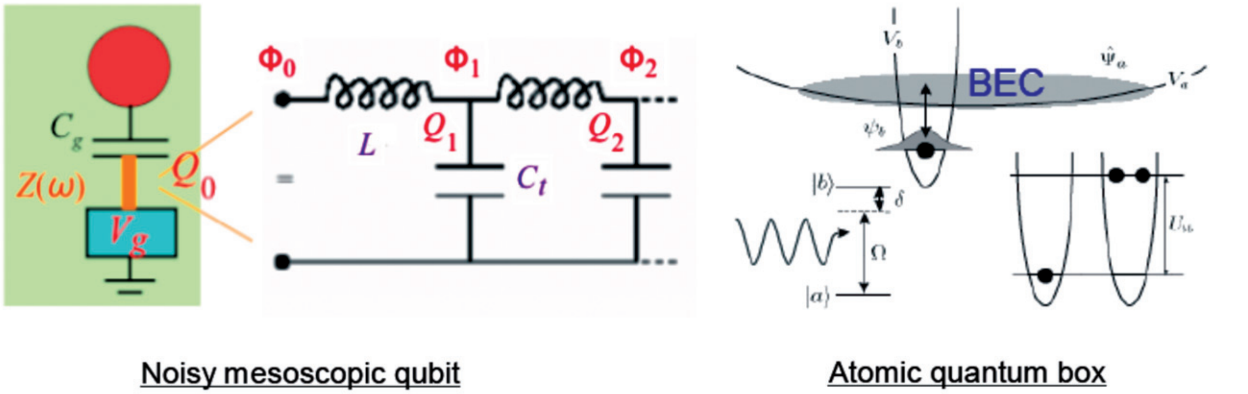

Figure 3. Possible realizations of the spin-boson model: the noisy mesoscopic qubit and the cold-atomic quantum box coupled to a Bose–Einstein condensate via coherent laser transitions and collisions [25,26]. (The color version of this figure is included in the online version of the journal.)

a persistent current which is proportional to $\langle S_x \rangle$ [9]. Solid-state two-level systems usually feature a coupling strength much below $\alpha^* = 1/2$. A very promising candidate in this sense (see Figure 3) is the ultracold-atomic quantum dot [25], which allows an *unprecedented* control of the coupling(s) between the qubit and the bosonic reservoir. This cold-atomic setup can be easily extended to the case of several qubits [26]. The (Ohmic) spin-boson model can also be engineered in trapped ions arranged in Coulomb crystals [27].

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