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Conceptualizing and Measuring Structural Inequality

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Abstract

This paper conceptualizes inequality in a society as having two basic types—individual inequality and structural inequality—with the former generated by individual variations whereas the latter responsive to social structure represented by social groups. Based on Blau's idea of the intersection between heterogeneity and inequality, we propose some structural inequality measures, including a disaggregative and an aggregative version of the structural Gini as well as a decomposable set of structural Theil measures and a standard error estimator for the structural Theil, as particularly useful implementations. The proposed measures are examined through one simulation and three empirical applications. The simulation study analyzed a range of two-class situations of varying degrees of inequality. Three empirical data analyses using the 1991 British Household Panel Survey, the 2005 Panel Study of Income Dynamics from the US, and the 2000 Luxembourg Income Study data from six EU countries are conducted. Used together with the conventional measures, the structural inequality measures provide a complementary picture of the form of inequality in a society; standing alone, these measures give a single index of the degree of stratification in the form of structural inequality glossed over by conventional measures.

Keywords:

Inequality, Heterogeneity, Gini Index, Theil Index, Stratification, Social Structure

Introduction

Inequality must be one of the most fundamental, most salient, and most fertile research topics in sociology and in the broader social sciences. However, there appears to be a paradox in studying inequality, be it reward-based such as income or health-related such as life expectancy. On the one hand, the literature teems with examples of treating populations and societies as single units measured by conventional inequality measures such as the Gini and the Theil statistics, many of which may be excellent research treatises in their own right by noted or budding social scientists alike (e.g., Becker, Philipson, and Soares 2005; Lee 2005). On the other hand, many scholars have repeatedly reminded us of persistent social inequality existing *between* social groups *within* a society or population that is, for example, gender based (e.g., Davis and Joshi 1998; McCall 2001; Reskin 1993) or race and ethnicity based (e.g., McCall 2001; Sandefur and Pahari 1989; Waters and Eschbach 1995).

This paper deals with the paradox by conceptualizing inequality exiting in social structures, in particular, between social groups in society, and by proposing new measures for the concept of social group-based "structural inequality". True, researchers have been aware of heterogeneity within the world and within a country, applying decomposition to study such heterogeneity for nations or blocks of nations (Bourguignon and Morrison 2002; Firebaugh 1999, 2000; Schultz 1998) or for social groupings such as gender, race/ethnicity, or social class (Jenkins 1995; Schultz 1998; Yitzhaki 2002). The ongoing empirical research using decomposition methods on inequality between social groups have been largely driven by original research done by, for example, Dagum (1997, 1998) and Shorrock (1980, 1982, 1984) and facilitated in the last decade or so by Jenkins's Stata module INEQDECO (1999) and more recently, his related modules of INEQRBD (2008) and DSGINIDECO (2009).

On a theoretical level, the proposed way to study structural inequality is related to the conventional ANOVA approach to inequality decomposition summarized by Firebaugh

(1999), and to the literature on polarization (see Duclus, Esteban, and Ray 2004; Yitzhaki forthcoming). The ANOVA approach states that the total inequality or variance is equal to the sum of between-group and within-group variances. The fundamental focus by scholars studying polarization also is on the relative contribution of groups. In either case, however, total inequality is defined in terms of individual variations in the entire population (or sample) whereas the concept of structural inequality in this paper concerns only individual variations between structural groups. Pair-wise individual variations in the same group are of no theoretical concern except that group-specific spreads, a function of within-group variations, are regarded as structure-based.

It is not the purpose of this paper to replace the useful inequality measures and methods based on the concept of polarization and on ANOVA type decomposition, which will continue to benefit empirical research. Rather, the aim here is to offer an alternative view on understanding and measuring inequality, a view that is based on *structural inequality*—the amount of inequality generated by social structural differentiation as opposed to the amount of overall individual variations including the ignorable type not responsive to social structure. Unlike decomposition methods, which break up conventional inequality indexes into components, structural inequality measures present in a single statistic the amount of inequality explained by the heterogeneous social structure. Furthermore, as it will become clear later, there are two components of structural inequality—social groups differ en masse in rewards and attributes and social groups differ in their within-group heterogeneity in rewards and attributes. Only the aggregate, group mean-based heterogeneity is captured by the ANOVA-type decomposition methods.

Why would one have to be bothered with structural inequality measures? In a perfectly equal or an entirely unequal society, such measures are not necessary because conventional measures can accurately reflect these conditions. In today's world, almost all societies have some structural foundations underlying their social inequalities, ignoring these structural influences would provide a downward bias in the assessment of the amount of inequality in society.

To anticipate the conclusion and to motivate the introduction of the concept of structural inequality, let us consider a simple example.[1] Suppose that a small population has four people, two men and two women. The two men's incomes are (10, 10) and the two women's incomes are (18, 2). Since the sex-specific means are both 10, conventional between-group inequality measures would result in zero. However, it is obvious that the sexes do not receive equal treatment in this society. For this population, the conventional Gini index gives a value of 0.30, attributed by between-individual differences; the structural Gini index to be presented in this paper gives a value of 0.40, attributed by between-sex individual differences, and is 1/3 increase over the conventional Gini. The conventional Theil's first measure gives a value of 0.18, and when decomposed, is entirely composed of within-group inequality. The structural Theil proposed in this paper presents an inequality level of 0.11 (measured on the same scale as the original Theil) that captures the between-sex income dispersion differential. Such a small population is of course unrealistic. If we increase the size by having a sample of 1,000 men and 1,000 women with the same kind of income distribution (i.e., all men earn 10 and half of the women earn 2 and the other half, 18), then we may assess the statistical significance of the structural Theil estimate, which has a standard error of 0.03, suggesting a result statistically significant at the 0.001 level at least. To overcome the inability of conventional methods in capturing such structural differences, this paper introduces the concept of structural inequality and a version of the structural Gini and structural Theil measures as well as the decomposition of the structural Theil statistic and its variance estimator for conducting significance testing.

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[1] The example is taken from Blackorby, Donaldson, and Auersperg's (1981) introduction of their procedure.

In the section following this introduction, we begin with a theoretical examination of the concept of structural inequality. We then discuss two modified conventional inequality measures—the structural variants of the Gini and the Theil indexes, with the latter having two subtypes (though the Theil structural subtypes are identical to each other). We also consider a variance estimator for providing standard errors for the structural Theil estimates. A simple simulation study illustrates the behavior of the structural inequality measures though here the interest is not in the statistical properties of the measures such as the relation between bias and efficiency on the one hand and sample size on the other because small samples are typically inappropriate for capturing inequality. We then demonstrate the use of these structural inequality measures with empirical data from the 1991 British Household Panel Survey (BHPS), the 2005 Panel Study of Income Dynamics (PSID), and the 2000 Luxembourg Income Study data from six EU countries, and the 2006 U.S. General Social Survey (GSS) before offering some concluding remarks.

The Concept of Structural Inequality

As indicated at the outset of the paper, the social science literature, regardless of the sociological or economic focus of the research, is predominantly concerned with inequality generated or at least related to some form of social structure, most notably gender, race/ethnicity, and class. However, conventional inequality indexes are routinely applied to the entire society or population, measuring individual variation in it, thereby ignoring structural inequality. Even though decomposition methods have been applied by researchers, that a single index, especially the Gini index, gets used every day and is compared across nations is especially troublesome if no attention is paid to structural inequality.

Recent scholarship suggests that the separation between individual-based and social group-based inequality is indispensable. Liao (2006) demonstrated the insensitivity of the

Gini ratio to stratified distribution of attributes and its sensitivity to differences in individual values, and defined two types of inequality—individual inequality, which is measured by differences between individual values, and class inequality, which is the degree of stratification, or put differently, how rank-based social groups differ from one another in terms of a certain attribute or reward.

In a similar vein, Jasso and Kotz (2008) identified two types of inequality—inequality between persons and inequality between subgroups. Their "personal inequality" "refers to inequality in the distribution of X in a set of person" and "subgroup inequality" "refers to the discrepancy between the distribution (most often the mean or other measure of central tendency) of X between two subgroups of a group (or population)" (p. 32). They gave examples of gender, nativity, race and religions as examples of subgroup inequality.

Comparing the two sets of concepts, we see that individual inequality and personal inequality are more or less the same; that is, they refer to pair-wise or overall differences existing among individuals in a distribution of goods and attributes. Whereas class inequality and subgroup inequality appear to emphasize different types of sources to which differences may be attributed, they are tied together by a common concern with some representation of social grouping—be it a relatively fixed type such gender, race, and religion or a relatively flexible kind such as social class where mobility can be observed. Merging these two definitions, we define *structural inequality* as the degree to which social groups such as race, gender, and class differ—group-wise or by individual members between groups—in terms of rewards and attributes, such as income, wealth, and health.

Needless to say, social groups that are relatively fixed are easy to operationalize because they are recorded on most social surveys. Social class is a different matter; it can be analyzed through occupation, to be reviewed below, or through the means of statistical analysis. This can be done through a multidimensional analysis because inequality that sociologists and economists are concerned about is, after all, multidimensional (Grusky and Kanbur 2006; Jenkins and Micklewright 2007). Such approach represents an extension of Liao's (2006) model-based estimation of social class to a multidimensional space and a realization of Grusky and Weeden's (2008) proposal of measuring poverty and inequality with the latent class model.

With no intention to offer a complete review of the literature on social class, we now turn to a highlight of the extant literature on social class, typically through the means of occupation or occupation-related job-holding, just to get a sense about how social class has been theorized. In North America, an influential theory on social class in late 20th century can be found in Wright's (1979, 1985) six-class scheme: the classical classification a la Marx—bourgeoisie (capitalists who employ others), petty bourgeoisie (self-employed who do not employ others), the proletariat (workers who neither own nor control their means of production)—plus Wright's additions of managers, supervisors, and autonomous workers. Responding to those theorists who use aggregated units of class, especially the Marxian or neo-Marxian type, Grusky and Sørensen (2001) proposed to think along the lines of disaggregation: By maintaining class analysis but shifting to the unit of analysis to much smaller "occupational associations" which actually exist in society, thus operating from a post-Durkheimian standpoint that views structuration at a disaggregated level.

In Europe, the more dominant stratification tradition can be summarized by Goldthorpe's 11-class schema in the three clusters of employers, self-employed workers, and employees (Erikson and Goldthorpe 1992). The employers can be of large or small firms, self-employed workers may work in industry or agriculture, and employees can be divided according to the form of regulation of employment, such as service relationship, intermediate, and labor contract. These divisions give rise to the influential seven categories represented by Roman numerals I to VII, with the IV to VII classes subdivided into subclasses such as IIIa, IIIb, etc., to VIIa, and VIIb (see Evans 1992, Figure 1).

Whether the source of stratification is social class, gender, or race/ethnicity, the type of inequality in a society that is so structured is lost in just focusing on individual inequality. The concept of structural inequality closely relates to that of social stratification as defined by Giddens (2006: 296) because it is "structured inequalities between different groupings of people." These groupings can be of a pre-given type ("pre-" in the sense of prior to a study or analysis) as gender, race/ethnicity, nativity, religious affiliation, occupation, social class as defined by the schemata discussed thus far, or the result of estimation by statistical means such as latent class models or model-based clustering.

Although the simple definition of "structural inequality" may sound reasonable, we must describe the theoretical foundation of the concept before moving on to the measurement issues. At a theoretical level, the concept of "structural inequality" is based on Peter Blau's (1977) work on inequality and heterogeneity. Following up on Simmel's (1950) concept of social structure, Blau (1977: ix) was interested in the relation between social inequality and heterogeneity because "[a] fundamental characteristic of social structures is the degree to which various forms of inequality and heterogeneity intersect or the extent to which social differences along various lines are correlated." On the one hand, we have nominal structural parameters, which can be an inborn attribute like sex or an acquired attribute like occupation; on the other, we have graduated structural parameters that differentiate people in terms of status rank-order, income, wealth, education, and power.

For Blau (1977: 8-9), heterogeneity refers to a horizontal differentiation or population distribution between/among groups in terms of a nominal parameter whereas inequality refers to a vertical differentiation or status distribution of according to a graduated parameter. For Blau, the more parameters intersect, the greater the structural complexity in a society. Of fundamental interest here is the intersection between nominal (such as race and gender) and graduated (such as income and wealth) parameters. Blau (1977: 108) believed that "increasing status differences among groups depress both rates of association among groups, owing to the status differences, and among strata, owing to the group differences," and that "[t]he intersection of nominal by graduated parameters increases the probability of interpersonal conflict between members of different groups and strata" (114). Put differently, heightened level of inequality (or vertical differentiation in terms of status) between and across nominal social groups (at the horizontal level) may have the social consequences of segregation and conflict between members of different social groups because of reduced level of association between the groups.

Therefore, based on Blau's theory on inequality and heterogeneity, we define structural inequality as the differentiation of a vertical distribution of a reward or status attribute such as income and wealth between and across a horizontal distribution such as race, gender, social class, and national origin groups. This horizontal differentiation of vertical distributions may realize in two ways—it may differ in terms of the group-specific total (or geometrical mean) of an attribute or in terms of the group-specific spread of the attribute. The greater amount in such differentiations there is in a society, the greater degree the structural inequality therein.

Because our central concern is the degree to which a society is differentiated in terms of (vertical) status and award by (horizontal) social groupings , be they fixed social groups such as gender and race whose membership in which is almost impossible to change or more fluid social groups such as social class that allows mobility between classes either within or over generations, we aim to measure a scaled degree of comparison either between social groups as a whole or between members of such groups in terms of their income shares relative to their population shares. Indeed, the method of making income shares relative to population shares is at the heart of inequality measures such as the Theil indexes and the Gini ratio (Theil 1967).

As stated earlier, the relation between vertical differentiation and horizontal distribution may realize in two ways. Applying some theoretical statistical thinking, we quickly see that the two types of horizontal differentiations in vertical distributions exit—differentiations defined by *location* and by *spread* or *dispersion*. Conceptually, these two components of vertical differentiation should sum up to the total amount of vertical differentiation described by horizontal distributions.

To reiterate, it is important to go beyond "individual inequality" to understand and analyze "structural inequality". To further operationally define structural inequality in a population or society, we use income share to represent the proportion of rewards, goods, or desirable attributes *y* (or vertical distribution) belonging in a section of population or society out of the total amount found in a population, and we use population share (or horizontal distribution) to represent the proportion of this section of population out of the total population. Let I_h indicate the income share of group *h*, P_h, the population of group h, I_k, the income of group *k*, and P_k, the population share of group k; we may express a type of measurement for structural inequality M_{SI} conceptually as:

$$M_{SI} = w \times f\left(I_h, P_h, I_k, P_k\right) \qquad (1)$$

where the income share and population share of group *h* are compared to that of group *k* by some mathematical means in a function that depends on a particular definition of a structural inequality measure, and weight *w* is necessary to scale the measure to a desirable range of the real scale, such that $M_{SI} \in \mathrm{R}$ of [0,1] as in the case of the Gini or of [0,∞] as in the case of the Theil.

Six comments on (1) are in order: First, when income-share-to-population-share ratio $\frac{I_k}{P_k}$ is used, the ratio makes income or rewards comparisons between groups possible by "standardizing" income shares so that a comparison of income shares of different groups can now be done on equal footing. This type of ratio is at the heart of several existing inequality measures, and may and will be preserved for structural inequality measures as well. Second, the functional form of a particular measurement definition takes must follow an existing inequality statistic so that existing properties, such anonymity, mean invariance or scale independence, population independence, and the Pigou-Dalton principle or the transfer principle (i.e., if some income transfers from a richer to a poorer person, then inequality should fall), may, though not necessarily always, be preserved.[2] These properties are important for the statistical behavior of a measure. Third, a specific functional form leads to a particular inequality measure. For example, $f(I_h, P_h, I_k, P_k) = \left(\frac{I_k}{P_k} - \frac{I_h}{P_h}\right)$ leads to the well-known (aggregate) between-group Gini measure, and $f(I_h, P_h, I_k, P_k) = \left(\frac{I_k}{P_k} \Big/ \frac{I_h}{P_h}\right)$ gives rise to a structural Theil measure, to be defined later. Fourth, w should be chosen in such a way that the resulting measure will fall into the same valid range of values of an existing inequality measure that a corresponding structural inequality measure is set up to modify. A range of values identical to an existing measure is also useful for interpretation; for example, the [0,1] range of the popular Gini index is so intuitively appealing and simple; a comparison across populations becomes extremely straightforward. The [0,∞] range of the Theil, though not as intuitively simple, still renders a comparison across populations possible. Fifth, it is true, too, that if a structural inequality measure follows a particular functional form of an

[2] Depending on how groups are formed, the Pigou-Dalton or the transfer principle may not necessarily be preserved.

existing measure, then it is sensible for it to also adopt the same valid range of values so that its usage will resemble that of the existing inequality measure. The relative consistency with an existing measure should facilitate the application of the corresponding structural inequality measure. Sixth, it is theoretically reasonable to obtain an income-share-to-population-share comparison at the individual level, such as $\left(\frac{I_{ki}}{P_{ki}} - \frac{I_{hj}}{P_{hj}}\right)$ or $\left(\frac{I_{ki}}{P_{ki}} \middle/ \frac{I_{hj}}{P_{hj}}\right)$ so that the statistical comparison is conducted between individual *i* in group *k* and individual *j* in group *h*, each made relative to the relevant income share.

These characteristics lead to an understanding of the concept of structural inequality akin to that of inequality without considering any structural effects in a population in terms of whether there are components in the total amount of inequality. For example, we may conceptualize inequality described by $\left(\frac{I_{ki}}{P_{ki}} - \frac{I_{hj}}{P_{hj}}\right)$ or $\left(\frac{I_{ki}}{P_{ki}} \middle/ \frac{I_{hj}}{P_{hj}}\right)$ as a disaggregative measure of structural inequality, or total amount of such inequality, and inequality defined by $\left(\frac{I_k}{P_k} - \frac{I_h}{P_h}\right)$ or $\left(\frac{I_k}{P_k} \middle/ \frac{I_h}{P_h}\right)$ as an aggregative form of structural inequality, or between-group inequality in the more familiar language of inequality measure decomposition. As we will see later, for measures such as the Gini that is not cleanly additively decomposable, it may make more sense to speak of dissaggregative and aggregative structural inequality whereas for measures such as the Theil that is decomposable, it can be more desirable to define the three components of total, between-group, and within-group structural inequality.

Although the concepts of disaggregative and aggregative structural inequality are easy to understand because they simply describe the amount of inequality between structural units, either between pairs of individuals of these units, or between the units as a whole, a component view of structural inequality needs some elaboration. Obviously, total structural inequality is disaggregative, and describes all pair-wise comparisons between all members of different groups; between-group structural inequality tells us how unequal the groups are as a whole, ignoring any within-group variation comparisons; within-group structural inequality, then, gives the amount inequality attributable to the contrasts or differences between groups in terms of their within-group distributions or differentiations. In other words, *total* structural inequality constitutes two components—the amount due to overall structural differentiation in rewards and attributes *between* groups as a whole and the amount due to specific structural differentiation in rewards and attributes between groups in terms of their *within*-group or group-specific variations in inequality.

To contrast with conventional measures of individual inequality, which states that total inequality is equal to the sum of the inequality represented by individual variation *within* groups and the inequality represented by group variation *between* groups, according to the concept of structural inequality, total inequality is equal to the sum of the inequality represented by individual variation *between* groups and the inequality represented by group variation *between* groups. The key difference between the two approaches lies in the individual variation term, whether it is individual variation *within* groups or individual variation *between* groups. In the former case, such the total inequality variation is $f(\mu_k, \sigma)$ while in the latter case, the variation is $f(\mu_k, \sigma_k)$, contributable by the three sources of individual-based spreads within groups (though not directly compared and computed), group-mean differentials or group variations between groups, and group-specific dispersion variations.

Because within-group or intragroup inequality is taken into account in structural inequality, the definition of structural inequality has some affinity with the procedure proposed by Blackorby, Donaldson, and Auersperg (1981) that treats an inequality index as the identity of unity minus the ratio of the equally-distributed-equivalent income to the mean income, or $I(y) = 1 - \frac{\Xi(y)}{\mu(y)}$. The idea of "equally-distributed-equivalent income" is at the core of their formulation; it is the "level of income per head which if equally distributed would give the same level of social welfare as the present distribution" (Atkinson 1970, p. 250). The level of equally-distributed-equivalent income can be made group-specific, thus bringing intragroup inequality into the calculation of overall inequality. What differs between the Blackkorby-Donaldson-Auersperg method and the current approach is that the level of a group-specific equally-distributed-equivalent, albeit reflecting inequality within that group, must be set by way of an *r* parameter (which ranges from 1, where inequality does not matter, to -∞, where only the income of the poorest person counts in the social evaluation, commonly known as maximin) whereas with the current conceptualization of structural inequality, intragroup inequality affects the measurement of inequality through between-group pairwise comparison, either in income-share-to-population-share ratio difference or in ratio of such ratios.

The Measurement of Structural Inequality

The Gini Measurement

The Gini Indexes

Let $\pi = F(y_i)$ indicate the distribution for y_i, and let $\eta = F_1(y_i)$ represent the corresponding first moment distribution function. The relation between η and π, defined for $0 \leq y_i < \infty$, is the Lorenz curve, and relation can be denoted by $\eta = L(\pi)$. The Gini index can then be defined accordingly:

$$G = 1 - 2\int_0^1 L(\pi)d\pi$$

And can also be written as

$$G = -1 + \frac{2}{\mu_y}\int_0^{\infty} yF(y)f(y)dy$$

There exits numerous computational formulae for implementing the Gini calculation. Perhaps the most revealing one is that used by Dagum (1997, 1998) and Mussard, Terraza, and Seyte (2003):

$$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|}{2n^2\mu} \qquad (2)$$

All the various computation formulae will give you the same results. However, (2) demonstrates that what is really measured by the Gini index is a weighted average of pair-wise differences between the individual cases in the sample. It is the overall individual mean differences that matter, whether or not these individuals may fall into classes or clusters. Although recent research has shown that Gini's mean difference is a superior measure of variability for non-normal distribution (Yitzhaki 2003), the Gini index does not capture well the stratified nature of the data, a more important point for studying social and economic inequality.

In general, the decomposition of Gini indices into the between-class and within-class components when the classes are ordered can be used as a means to partition income inequality for the purpose of measuring stratification. For further elaboration on the Gini decomposition methods, see Dagum (1997). The overall Gini is computed as (2), and the within and between components of inequality for each subpopulation are computed respectively as below (Mussard, Seyte, and Terrasa 2003).[3] For the Gini within subpopulation *k*:

[3] Equation (4) is a corrected version of Equation (3) of Mussard, Seyte, and Terrasa 2003 where the denominator has only $(\mu_k + \mu_h)$, leaving out $n_k n_h$, an obvious mistake that would inflate the between-group Gini value. The cross product of $n_k n_h$ is an equivalent to the part of the usual Gini weight n^2 in (3).

$$G_{w\cdot k} = \frac{\sum_{i=1}^{n_k}\sum_{j=1}^{n_k}\left|y_i - y_j\right|}{2n^2\mu_k} \tag{3}$$

where n_k is the size of each k class or cluster, and k, a specific known or estimated classes or clusters, and the between-group Gini is defined as

$$G_{b\cdot kh} = \frac{\sum_{i=1}^{n_k}\sum_{j=1}^{n_h}\left|y_i - y_j\right|}{n_k n_h\left(\mu_k + \mu_h\right)} \tag{4}$$

where k and h are any pair of classes or group in the sample or population (Dagum 1987).

Now let us express the Gini index purely as a measure of structural inequality in a population or sample by using income share to population share ratios as a form of the function $f(I_h, P_h, I_k, P_k) = \left(\frac{I_k}{P_k} - \frac{I_h}{P_h}\right)$. Let $\sum_i y_i = s_t$, $\sum_i y_{ki} = s_k$ for group k with size n_k, and $\sum_i y_{hi} = s_h$ for group h with size n_h, we obtain:

$$G_{sa} = \frac{1}{2}\sum_{k=1}^{K}\sum_{h=1}^{H}\frac{n_k}{N}\frac{n_h}{N}\left|\frac{s_k/s_t}{n_k/N} - \frac{s_h/s_t}{n_h/N}\right| \tag{5}$$

As the reader can see, the G_{sa} index follows the conceptual setup of (1). In the present case, the functional form applied is the absolute difference between the income-share-to-population-share ratio of one group and that of another, and the one-half of a weighted average guarantees the resultant statistic follows into the [0,1] range. As it turns out, G_{sa} is actually identical to the between-component of the Gini decomposition mathematically (see Theil 1967).

For the Gini index, it is also possible to contrast each member of a particular group with the members of another group, in the same way as the usual Gini index in (2). However, now the absolute difference is calculated only between members of the contrasting groups, and the pair-wise group differences are summed over for all pairs of group comparisons:

$$
\begin{aligned}
G_{sd} &= \sum_{k=2}^{K}\sum_{h=1}^{k-1}\frac{n_k+n_h}{(K-1)N}\frac{1}{N\left(\frac{s_k}{s_t}\frac{n_h}{N}+\frac{s_h}{s_t}\frac{n_k}{N}\right)}\sum_{i=1}^{n_k}\sum_{j=1}^{n_h}\left|\frac{y_{ki}}{s_t}-\frac{y_{hj}}{s_t}\right| \\
&= \sum_{k=2}^{K}\sum_{h=1}^{k-1}\frac{n_k+n_h}{(K-1)N}\frac{\sum_{i=1}^{n_k}\sum_{j=1}^{n_h}\left|y_i-y_j\right|}{n_k n_h(\mu_k+\mu_h)}
\end{aligned}
\tag{6}
$$

The numerator of the first line in (6) gives the between-group member-wise absolute income-to-total-income-share differences and the denominator gives a common two-group population share (adjusted by income share), thereby also forming a between-group member-based income share to population share ratio. The weight in the denominator of the second line of (6) is identical to the weight for group-wise comparisons of the between-group Gini component weight of (4), and the first weight $\frac{n_k+n_h}{(K-1)N}$ in either line of (6) guarantees the computation of the Gini is properly weighted according to the relative group sizes when more than two groups are involved. The unique component Gini cumulated for the entire sample or population gives our second overall structural Gini, based on the computation of member-wise group contrasts for the entire sample or population. G_{sd} measures the total amount of structural inequality for all individuals in the structure.

Note that G_{sa} and G_{sd} are measured on the same scale as G, with the former always smaller than G because it is a part of G. When there is no within-group differentiation, G_{sd} and G_{sa} are identical; when there is, $G_{sd} > G_{sa}$. When there is no between-group difference, $G_{sa} = 0$ and G_{sd} = G. Another way to interpret G_{sd} and G_{sa} is given by components in vertical differentiation by horizontal distributions. G_{sd} contains contributions of both locational and dispersional variations whereas G_{sa} captures locational differences only because locational differences are mean differences and an aggregated income difference is given by differences in group means times group totals.

The Limits of $G_{s\cdot}$:

Because the limits of $G_{s\cdot}$ should be defined the same way as the between-component of the Gini decomposition, let us focus on the limits of $G_{s\cdot}$ here. The upper limit is given by the situation where one person has everything while all the others have nothing—the most severe inequality imaginable. When this is the case, if we let the first component *k* in the numerator involving the summed absolute differences in (5) or in the second line of (6) to represent the rich person and the second component, *h*, the poor, it follows that n_k=1, μ_k=the rich person's average *and* total income, n_h=n-1, and μ_h=0. Then both the numerator and the denominator in (5) reduce to $n_h\mu_k$. For (6), the last terms becomes the total income of group *k* divided by the same quantity.

From (5) or (6) it is apparent that the lower limit of the structural inequality of the Gini index is zero when there is 100% structural equality because the numerator after the summation signs in (5) or (6) is zero when the summed group-wise or member-wise differences are zero between all pairs of groups or members.

The Theil Measurement

The Indexes:

Another popular measure of inequality is the Theil index, attributed to Theil (1967). There are actually two Theil's measures, the Theil's T or Theil's first measure, and the Theil's L or Theil's second measure. The Theil's measures are special cases of the class of generalized entropy measures. The concept of entropy traces its roots back to Shannon (1948) in his formation of information theory (also see Rathie 1970). In a finite discrete probability distribution of $P = \{p_1, p_2, \ldots p_N\}$, the entropy of the distribution *P*, or the expected amount of information in a distribution, is defined as

$$H(P) = -\sum p_i \ln p_i$$

and $\sum p_i = 1$.

Theil's (1967) ingenious application of the entropy concept to the study of inequality has profound implications. To see the point, the classic Theil's measure, Theil's T is given by

$$T_{Tt} = \sum_{i=1}^{N} \frac{y_i}{\sum_i y_i} \ln \frac{y_i / \sum_i y_i}{1/N} \tag{7}$$

Essentially, what Theil did was to replace p_i with the ratio of income share to population share, and weight the resulting ratio by income share. The negative sign is no longer necessary because $-\ln(p_i)=\ln(1/p_i)$. Theil's ratio in $\ln(\cdot)$ can be greater or smaller than 1, but the weight of income share guarantees that the summation ends in a value in the range of $[0,\infty]$.

As noted earlier, it is common to decompose Theil's index into its between- and within-components. Using the same notation, Theils's T can be decomposed into a between-component

$$T_{Tb} = \sum_{k=1}^{K} \frac{s_k}{s_t} \ln \frac{s_k / s_t}{n_k / N} \tag{8}$$

And a within-component of

$$T_{Tw} = \sum_{k=1}^{K} \frac{s_k}{s_t} \sum_{i=1}^{n_k} \frac{y_{ki}}{s_k} \ln \frac{y_{ki} / s_k}{1/n_k} \tag{9}$$

where we again use the shorthand of $\sum_i y_i = s_t$ for the sum of the entire sample or population and $\sum_i y_{ki} = s_k$ for group k with size n_k.

We now continue in the same specification of (7), except this time we replace the odds in the numerator of Theil's $\ln(\cdot)$ term with an odds ratio, and the odds in the denominator in (7), too, with an odds ratio because we now focus on structural inequality represented by $f(I_h, P_h, I_k, P_k) = \left(\frac{I_k}{P_k} \Big/ \frac{I_h}{P_h} \right)$ instead of just $\frac{I_k}{P_k}$ for (8); the same logic works for (7) and (9) as well. Each of these two odds ratios are equivalent to what is in Theil's ln(·) mathematically except now the top odds ratio represents the I_k to P_k (again, I_k and P_k refer to the income share and the population share for group k) ratio while the bottom odds ratio, the I_h to P_h ratio. Identical to Theil's formulation, the top cross-product in the numerator from the second line in (10) is used as the new weight, in a similar fashion to the top probability (or income share) is used as the weight in T_{Tt}. Therefore, just like T_{Tt}, the structural Theil's T total, albeit in odds-ratio ratio, is also an entropy measure that follows the same information theory principle, and it can be expressed as:

$$T_{sTt} = \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{ki=1}^{n_k}\sum_{hj=1}^{n_h}\frac{y_{ki}}{2s_t N}\ln\left(\frac{\frac{y_{ki}/s_t}{1/N}}{\frac{y_{hj}/s_t}{1/N}}\right) \qquad \text{for } k\neq h \qquad (10)$$

$$= \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{ki=1}^{n_k}\sum_{hj=1}^{n_h}\frac{y_{ki}}{2s_t N}\ln\frac{(y_{ki}/s_t)(1/N)}{(1/N)(y_{hj}/s_t)}$$

In addition to the probability of $\frac{y_{ki}}{s_t N}$ from the numerator of the line 2 in (10), we also need as part of weight ½ because each group appears in (10) as k and h, the reason for its appearance in both the numerator and the denominator being for preserving the original property of the Theil as an entropy measure. Similarly, based on (8), the between-component, or the aggregative version of the structural Theil is defined as:

$$T_{sTb} = \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_k n_h}{2s_t N}\ln\frac{\frac{s_k/s_t}{n_k/N}}{\frac{s_h/s_t}{n_h/N}} \qquad \text{for } k\neq h \qquad (11)$$

$$= \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_k n_h}{2s_t N}\ln\frac{(s_k/s_t)(n_h/N)}{(n_k/N)(s_h/s_t)}$$

Again following the same principle for defining (10) and (11), we further modify (9) and define the within-component of the structural Theil measure as:

$$T_{sTw} = \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_h n_k}{s_t N}\sum_{ki=1}^{n_k}\sum_{hj=1}^{n_h}\frac{y_{ki}}{s_k n_h}\ln\left(\frac{\frac{y_{ki}/s_k}{1/n_k}}{\frac{y_{hj}/s_h}{1/n_h}}\right)$$

$$= \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_k n_h}{2s_t N}\sum_{ki=1}^{n_k}\sum_{hj=1}^{n_h}\frac{y_{ki}}{s_k n_h}\ln\frac{(y_{ki}/s_k)(1/n_h)}{(1/n_k)(y_{hj}/s_h)} \quad \text{for } k \neq h \qquad (12)$$

$$= \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{ki=1}^{n_k}\sum_{hj=1}^{n_h}\frac{y_{ki}}{2s_t N}\ln\frac{(y_{ki}/s_k)(1/n_h)}{(1/n_k)(y_{hj}/s_h)}$$

It is important to note that (10), (11), and (12) give a measure on the same measurement scale as those in (7), (8), and (9), and preserve the same statistical properties as entropy measures. We will demonstrate later in the paper that in extreme circumstances, (10), (11), and (12) may even give identical values as (7), (8), and (9).

Notice that (11) and (12) sum up to (10) and are the two components of (10). Put differently, (10) records the total amount of structure-based inequality, (11) captures the amount attributed to locational differences, and (12) reflects the amount explained by dispersional differences. Total structural inequality can be decomposed into two additive components—a location-based component and a dispersion-based component.

Theil's second measure, or Theil's L, follows the same mathematical logic as his first measure, except the roles I_k and P_k play are now reversed:

$$T_{Lt} = \sum_{i=1}^{N}\frac{1}{N}\ln\frac{1/N}{y_i/\sum_i y_i} \qquad (13)$$

Note that T_{Tt}, or Theil's first measure, and T_{Lt}, or Theil's second measure, are both members of the class of generalized entropy inequality measures when the parameter (often indicated by α) approximates 1 for T_T and 0 for T_L. Without complication, the between-component and the within-component of Theil's second measure can be defined accordingly in the same fashion as (8) and (9), thus we omit them here.

Following the same formulation as for the structural Theil's T, we obtain the between-component structural Theil's L and express it as:

$$T_{sLb} = \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_h n_k}{s_t N}\ln\frac{\dfrac{n_k/N}{s_k/s_t}}{\dfrac{n_h/N}{s_h/s_t}} \quad \text{for k≠h} \qquad (14)$$

$$= \sum_{k=1}^{K}\sum_{h=1}^{H}\frac{s_h n_k}{s_t N}\ln\frac{(n_k/N)(s_h/s_t)}{(s_k/s_t)(n_h/N)}$$

Some readers may have realized by this point that a component of the structural Theil's T and is mathematically identical to its counterpart of the structural Theils L. The reason for the identity is because we have pair-wise comparisons in terms of log-odds ratios of any of the two groups out of a total of *G* groups. Suppose we have a simple two-group situation, where *k*=1 and *h*=2. It follows that (9) leads to (14):

$$T_{sTb} = \frac{s_k n_h}{s_t N}\ln\frac{(s_k/s_t)(n_h/N)}{(n_k/N)(s_h/s_t)} + \frac{s_h n_k}{s_t N}\ln\frac{(s_h/s_t)(n_k/N)}{(n_h/N)(s_k/s_t)} = T_{sLb} \ \text{for k≠h} \qquad (15)$$

This can be extended to analyses with multiple groups where G>2 with more than one unique group-wise contrast. Thus, we may simplify the subscript by using just T_{sb} (and T_{st} and T_{sw}) hereafter, dropping the *T* and *L* part of the subscript. For the same reason, we do not present the formulation for the Theil L counterpart of the total or the within-component because they are identical to (10) or (12).

Limits:

The lower limit for either $T_{s\cdot}$ in (10), (11), (12) or (14) is reached when there exists 100% structural equality, that is, the income share of each group or member equals to its population share, resulting in a lower limit of 0 because ln(1/1)=0.

The upper limit is just slightly more involved to figure out than the lower limit. Imagine there are just two social groups for, say (11), with the first group (group *k*) having an extremely small proportion of the population (or sample) but all the income and the second group having none of the income but an extremely large proportion of the population (or sample). As a result, the numerator in ln(·) in (a component of) the structural Theil → ∞ when the population share of group k p_k→0, and the denominator in ln(·) → 0 when the population share of group h p_h→1, resulting a ln(·) term → ∞ for group k weighted by a term→1; this is summed with the result when the second group takes over the position of k, resulting in a ln(·) term → 0 weighted by a term→0. Similar steps can be worked for (10) and (12). Therefore, the upper limit for $T_{s\cdot}$ in (10), (11), or (12) is positive infinity. Similarly, the upper limit for T_{sb} in (14) or its related within-component or the total structural Theil is also positive infinity, a trivial case to prove because by simply switching the positions of the first and the second group, now the second group having all income while the first having almost all the population. The only thing changed is that now the second term before summation is the one with a value → ∞, resulting again, a $T_{s\cdot}$ → ∞.

A variance estimator:

Because the core component of the structural Theil is a log-odds ratio, we adapt a standard error computation for log-odds ratios commonly applied in social science and epidemiological research (see Agresti 1999 for further details, who also proposed an estimator for small samples); the estimator log-odds ratio $\ln\hat{\theta}$ involving the four frequencies of n_{11}, n_{12}, n_{21}, and n_{22} is approximately normal with asymptotic standard error estimable by

$$\hat{\sigma}\left(\ln\hat{\theta}\right)=\sqrt{\frac{1}{n_{11}}+\frac{1}{n_{12}}+\frac{1}{n_{21}}+\frac{1}{n_{22}}} \qquad (16)$$

where n_{ij} is the observed frequency for the i=1, 2 and j=1, 2 cells of a 2 by 2 (sub)table in a series of 2 by 2 tables. The statistical problem of developing a variance estimator for a pooled log-odds ratio is that for the Mantel-Haenszel statistic. Of all various proposals for a variance estimator, we follow Morris and Gardner's (1988) approach that extends the principle of (16) to the pooled case where the square-rooted variance estimator s is formulated as $s = \sqrt{\dfrac{1}{1/n_{11} + 1/n_{12} + 1/n_{21} + 1/n_{22}}}$. The obtained standard error can either be used for conducting a standard significance test or for forming an approximate 100(1-α)% confidence interval by the endpoints of $\ln\hat{\theta} \pm z_{\alpha/2}\hat{\sigma}(\ln\hat{\theta})$ with $z_{\alpha/2}$ being the $\alpha/2$ standard normal quantile. However, this idea cannot be applied to the structural Theil statistics of (10), (11), (12), or (14) directly because the log-odds ratio therein is formed by ratios of probabilities or relative frequencies instead absolute frequencies. Some adaptation is necessary.

We first rearrange terms in both the numerator and the denominator part of the top line in (10), (11), and (12) so that relative frequencies of the four entries of the odds ratio are turned into absolute frequencies or values. We then weight each contribution of a single summation of the inverse of the four component frequencies by the same weight as in (10), (11), and (12) so that the components of a "summary odds-ratio" are computed before they are raised to the power of 0.5 and multiplicatively inversed, obtaining an standard error estimator for the total, the between-component, and the within-component of the structural Theil measure:[4]

$$\hat{\sigma}(T_{st}) = \left(\sqrt{ \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{ki=1}^{n_k} \sum_{hj=1}^{n_h} \frac{y_{ki}}{2Ns_t} \left(\frac{1}{Ny_{ki}} + \frac{1}{s_t} + \frac{1}{Ny_{hj}} + \frac{1}{s_t} \right)^{-1} } \right)^{-1} \quad \text{for } k \neq h \tag{17}$$

$$\hat{\sigma}(T_{sb}) = \left(\sqrt{ \sum_{k=1}^{K} \sum_{h=1}^{H} \frac{n_h s_k}{2Ns_t} \left(\frac{1}{Ns_k} + \frac{1}{n_h s_t} + \frac{1}{Ns_h} + \frac{1}{n_k s_t} \right)^{-1} } \right)^{-1} \quad \text{for } k \neq h \tag{18}$$

and

[4] An alternative method of directly applying an adapted version of (16) without using the inverse-inverse function of Morris and Gardner (1988) was considered. Simulation results show that the version implemented in (17), (18), and (19) provide higher sensitivity to changes in the three sources of structural-within variation, especially in dispersion disparity between groups.

$$\hat{\sigma}(T_{sw}) = \left(\sqrt{ \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{ki=1}^{n_k} \sum_{hj=1}^{n_h} \frac{y_{ki}}{2Ns_t} \left(\frac{1}{n_h y_{ki}} + \frac{1}{s_k} + \frac{1}{n_k y_{hj}} + \frac{1}{s_h} \right)^{-1} } \right)^{-1} \quad \text{for } k \neq h$$

(19)

where in (17), (18), and (19) the calculation of summing over the groups is performed in the same way as in (10), (11), and (12). The obtained standard error can then be further used in forming confidence intervals or performing significance tests.

Following (16), we can easily show that the square-rooted variance estimators for the *T* and *L* forms of the structural Theil coefficient for a two-group contrast are, too, mathematically identical. Take, (18), for example:

$$\hat{\sigma}(T_{sTb}) = \left(\sqrt{ \frac{n_h s_k}{2Ns_t} \left(\frac{1}{n_k s_h} + \frac{1}{Ns_t} + \frac{1}{n_k s_h} + \frac{1}{Ns_t} \right)^{-1} + \frac{n_k s_h}{2Ns_t} \left(\frac{1}{n_h s_k} + \frac{1}{Ns_t} + \frac{1}{n_h s_k} + \frac{1}{Ns_t} \right)^{-1} } \right)^{-1} = \hat{\sigma}(T_{sLb})$$

for k≠h (20)

The two-group case, once again, can be generalized to multiple-group scenarios where *G*>2. Therefore, hereafter we drop the subscript *T* or *L* to use just $\hat{\sigma}(T_{s\cdot})$.[5]

One may wonder why the original Theil makes a distinction between the T and L forms whereas the structural Theil does not. The reason for the difference lies in the difference between log-odds and log-odds ratios. In all the original Theil formulations, regardless of whether it is the total or its components, two different groups never appear in the same log-odds simultaneous whereas in (10), (11), (12), and (14) they do appear in the same log-odds ratio at the same time, thereby providing identity between the two forms. Because of this characteristic, each group appears twice, both in the numerator and in the denominator, thereby needing to be divided by 2. Therefore, the structural Theil measures can be considered as a symmetrized Theil index.

[5] Preliminary simulation studies demonstrate that these analytic standard error methods are more sensitive to changes in data than bootstrap or jackknife type of standard errors.

Furthermore, the structural Theil measures are measured on the same measurement scale as the original Theil measures. This point can be well illustrated by applying both the original Theil component measures and their structural counterparts to six simple cases of inequality that are best described by the Pareto principle plus an extreme case of inequality. That is, the population has the following percentage distributions: (1) a 50.0:50.0 distribution that is represented by a Theil index with a value of 0; (2) a 74.0:26.0 distribution that gives a Theil value of 0.5; (3) a 82.4:17.6 distribution that is described by a Theil value of 1; (4) a 91.7:8.3 distribution that can be captured by a Theil index equal to 2; (5) a 98.4:1.6 distribution that stands for a Theil value of 4. (6) a 99.9:0.1 distribution that can be represented by a Theil value of close to 7. We present the original Theil measures, the structural Theil measures, alongside the Gini measures, in Table 1.

---Table 1 about here---

As is clearly shown in the table, all these are trivial cases, which nonetheless illustrate the properties of the measures well. First of all, when the population has two groups within which there is no individual variation, the between-component Theil will be equal to the total Theil inequality, with the within-components equal to 0, and the G and G_a for the same situation are all identical (though the G_d are larger for datasets D_2 to D_6 because within-group pair-wise individual differences, which are all zeros in the current situations, are not part of the computation). Second, in these situations described by the Pareto principle, the Theil T and the Theil L measures are no different from each other. Third, the structural Theil measures are indeed on the same scale as the original Theil, and give the same values in these simple inequality scenarios. In fact, when the number of groups is equal to the number of cases, T_{st} is located half-way between T_{Tt} and T_{Lt} or $T_{st}=(T_{Tt}+T_{Lt})/2$ because each person has his/her own group; when there is no within-group heterogeneity, it is tantamount to having equality between number of groups and cases and $T_{Tt}=T_{Lt}=T_{st}$ as shown in Table 1. Fourth, the standard errors of the between-group structural components are rather small (less than ½) as compared to the between-group estimates in data situations of D_2 to D_6; but neither the total nor the within-group component standard errors are small enough because the total is strongly influenced by the within-group component. Fifth, the structural Theil measures will only be equal to the original Theil when there is no within-group heterogeneity but will not be equal to the original Theil measures when there exist within-group inequalities, as to be shown by the simulated and the empirical examples in the next section. Finally, these six data scenarios establish some nice benchmarks for gauging inequality on the $[0,\infty]$ or $[0,1]$ scale. Interested readers may want to generate additional benchmarks for guiding interpretation of the Theil type of measures, or may want to compute benchmarks that facilitate the interpretation of the Gini if necessary (i.e., the value in income-share/population-share split that corresponds to G=0.1, 0.2, 0.3, …, 0.9).

Applications

A Simulation

To get a sense of how these structural inequality measures behave, we present a simulation study stratified by two equal numbered classes (such as men and women), considering a reasonable range of inequality as observed in society. We set up 15 simulated income distributions in sets of three, all containing two classes of 1,000 cases each. The first distribution in any set of three has no distinction whatsoever between the two classes in terms of their income distribution; the second distribution in a set has the two sub-distributions representing the two classes overlap approximately 25% (anywhere between 20% to 30%), i.e., the top quartile of the first group overlaps with the bottom quartile of the second group; the third distribution in a set has the two classes clearly separated, i.e., the top income-earner of Group 1 has a lower value than the bottom income earner of Group 2. Therefore, all five sets have identical setup in terms of structural inequality.

The five sets of distributions constitute five levels of (individual) inequality found in countries of the world, as judged by the middle distribution of each set that has the 25% overlap. That is, the second distribution in Set A has a Gini index in the 10-20% range; the second distribution in Set B has a Gini index in the 20-30% range; Set C, in the 30-40% range; Set D, in the 40-50% range; and Set E, in the 50-60% range. The 15 formulas below are the random probability generation functions of two mixing log-normal distributions for each of the two classes in an overall distribution (with mean and standard deviation as the two parameters in the parentheses):

$$y_{A1i} \sim \log N(2.0,0.1), \log N(2.0,0.1),$$

$$y_{A2i} \sim \log N(1.8,0.1), \log N(2.2,0.1),$$

$$y_{A3i} \sim \log N(1.6,0.1), \log N(2.4,0.1),$$

$$y_{B1i} \sim \log N(3.0,0.2), \log N(3.0,0.2),$$

$$y_{B2i} \sim \log N(2.6,0.2), \log N(3.4,0.2),$$

$$y_{B1i} \sim \log N(2.2,0.2), \log N(3.8,0.2),$$

$$y_{C1i} \sim \log N(4.0,0.3), \log N(4.0,0.3),$$

$$y_{C2i} \sim \log N(3.4,0.3), \log N(4.6,0.3),$$

$$y_{C3i} \sim \log N(2.8,0.3), \log N(5.2,0.3),$$

$$y_{D1i} \sim \log N(5.0,0.4), \log N(5.0,0.4),$$

$$y_{D2i} \sim \log N(4.2,0.4), \log N(5.8,0.4),$$

$$y_{D3i} \sim \log N(3.4,0.4), \log N(6.6,0.4),$$

$$y_{E1i} \sim \log N(6.0,0.5), \log N(6.0,0.5),$$

$$y_{E1i} \sim \log N(5.0,0.5), \log N(7.0,0.5)$$, and

$$y_{E1i} \sim \log N(4.0,0.5), \log N(8.0,0.5)$$.

To obtain a visual sense of the distributions, the histograms of the simulations are reported in Figure 1. Because of the lognormal distribution, identical standard deviations actually mean differential spreads on a non-log scale between the two groups especially when they are further part. These simulations are one-pass only because the assessment of asymptotic efficiency of a certain estimator with regard to sample size is not the goal here. Rather, the chief purpose is to see how various inequality statistics compare.

---Figure 1 about here---

Clearly, all distributions are skewed due to the lognormal distribution generating function, as income distributions in reality should. Moving in rows from A to E, the distributions become more spread out and skewed. In the first column, the distributions show no separation at all between the two classes; in the second column, they show some separation as well as some overlapping; in the third column, there is clear separation between the two subsets of cases in the distributions.

Some general observations are in order. First, judged by the classic Gini ratio G, the inequality of the simulated datasets ranges from a low of 0.05 for A_1 to a high of 0.62 for E_3, and the second of each simulation sets (i.e., A_2, B_2, C_2, D_2, or E_2) does fall in the expected range of values (i.e., 10-20%, …, 50-60%) as described earlier (Figure 2). We did not expand to total inequality as it has not been observed in today's countries of the world.

---Figure 2 about here---

Second, as expected, the amount inequality represented by G_d or structural Gini is more or less equal to G for all the simulations when there is no group-based inequality, and larger when there is such inequality. The G_a values are always a portion of the G, and the structural Gini or G_d are all greater than G when there is structure-based inequality, especially with simulations A_3, B_3, C_3, D_3, and E_3. Whereas G_a gives a total, aggregative between-class component of G, G_d or structural Gini is influenced by both the location-based and the dispersion-based differentials. Therefore, G_d can be used in tandem with G (and and G_a) to get a sense of structural inequality when the overall amount of inequality is low. Used alone, one can determine how much dissaggregative structural inequality there is, as indicated by the simulations in Figure 2 (and by Figure 1).

Third, the Theil measures and their decompositions also provide an assessment of the amount of individual inequality in the income distributions. T_T, or Theil's first measure, and T_L, or Theil's second measure, increase their values when the amount of total individual inequality increases from distribution to distribution, with the second measure increasing at a faster rate for data with greater amount of inequality. The between and within decomposition measures give the proportion of the Theil indexes that are influenced by between and within group inequality (Figure 3).

---Figure 3 about here---

The $T_{s\cdot}$ or structural Theil measures present an interesting case because they clearly reveal structural inequality well. When there is no structural inequality, the total structural Theil or its component values are always close to 0 (for A_1, B_1, …, E_1); when structural inequality exists though not of an extreme amount, their values are at a higher level (for A_2, B_2, …, E_2); when structural inequality is severe with 100% separation between the classes, their values are much more heightened (for A_3, B_3, …, E_3). Just like the conventional Theil, the structural Theil index has a valid range of values from 0 to positive infinity, and they tend to fall between Theil's T and Theil's L values. We must note that it is incorrect to regard the structural Theil measures as a portion of the Theil measures even though they are measured on the same scale. The structural measures represent, on the same Theil scale, the amount of inequality that is entirely due to contrasts of members between (members of) different groups. As shown in Table 1, when all inequality is structural, the measures give identical values to those of the original Theil. When inequality is anywhere between the situation when there is no structural inequality and the one there is 100% structural inequality, the structural measures capture the amount of structural inequality on the same Theil scale, and are influenced by the group-specific dispersional differences, as we move from data situations A to E when dispersional differences increase.

Finally, applying the $\hat{\sigma}(T_{s\cdot})$ estimator to $T_{s\cdot}$ should further help us assess the degree of structural inequality. When there is little structural inequality, the standard error estimates are large relative to $T_{s\cdot}$; when there is significant structural inequality, the standard error estimates are small relative to $T_{s\cdot}$. Here we may apply the usual *t*-test to assess the degree of structural inequality, with $T_{s\cdot}=0$ as the null hypothesis, or construct confidence intervals for $T_{s\cdot}$. As indicated in the footnote to Figure 3, all the simulations with consequential differences in location and/or dispersion produced standard errors smaller than than ½ of the structural Theil estimates.

The 1991 British Household Panel Survey Example

Next, we apply the structural inequality measures to the 1991 BHPS household income data. The BHPS is an annual survey of each adult member aged 16 and over of a nationally representative sample of more than 5,000 households (approximately 10,000 individual interviews). We analyzed data from the 1991 panel because it was the first wave, before the survey lost any cases to panel attrition. The BHPS in 1991 had an original sample of 5,500 households. The BHPS records annual household income and also offers an equivalence scale for adjusting for household needs. By analyzing household income, we do not intend to generalize patterns to inequality based on individual income. We analyzed the British annual household income data three times, first without any adjustment, then needs-adjusted with regard to the size of the household, finally needs-adjusted with regard to the size of the household and housing cost. In the analysis presented below, we examine household income with the household head's gender as well as race as the grouping variable.

Whereas the sex/gender of the household heads presents a simple case of grouping, the race and ethnicity information from the survey is recoded into the five groups of whites, black Africans, Caribbean and Other blacks, Indians, and Pakistanis and Bangladeshis. The recoding was done to preserve the ethnic differences in their socioeconomic characteristics in Britain. Chinese and other ethnic groups were omitted due to the small sample size. There were altogether 3,710 male heads (mean income or μ=£19,190) and 1,584 female heads (μ=£9,207). The breakdown of headship by ethnicity is as follows: 4,946 whites (μ=£16,070), 50 Caribbean and other blacks (μ=£13,980), 23 black Africans (μ=£13,800), 47 Indians (μ=£20,910), and 25 Pakistanis and Bangladeshis (μ=£10,780).

Structural inequality in the BHPS income data is assessed three times—that is, income inequality without any adjustment and with the two types of adjustment on nonzero income values only because the number of zero cases (there is no negative values in the data) is not large and because including those cases would not increase total inequality much (as a preliminary analysis suggested). The results are presented in Table 2.

---Table 2 about here---

The more income is needs-adjusted by the equivalence scales, the greater amount inequality tends to decrease, and the amount of decrease is more sizeable for gender-related assessment than for race-related assessment. Furthermore, there appears to be a greater amount of gender-based structural inequality than there is race-based structural inequality, judged by either the structural Gini or the structural Theil measures.

How do we interpret the structural inequality measures? The G_d measures give us the amount of inequality that is accounted for by pair-wise comparisons between members of all different groups, thus reflecting both between-group locational and dispersional differentials, and the G_a values gives the amount that is attributed to the group comparisons as a whole or aggregated, the notion of the typical between-group component. There is definitely more structural inequality that is gender-based according to the structural Gini measure G_d.

The structural Theil statistics measure *only* the contrasts in terms of income-share to population-share log-odds ratios between all members between group pairs (the total structural Theil), the contrasts between pairs of groups as a whole in terms of the same ratios (the between-group component in terms of locational differences), and the contrasts between the ratios formed by individual income shares to population shares (the within-group component in terms of dispersional differences). Note that the between-component represents a much larger proportion out of the amount of total structure-based inequality that it is out of the amount of total individual-based inequality. The within-component is interesting because it sums up the total amount of within-group inequality variation between pairs of groups. Even though there is a smaller amount of race-based structural inequality than there is gender-based structural inequality, the amount still is statistically significant at all conventional type-1 error values, judged by the standard error estimates. Note that the significance level for testing racial contributions to income differentiations might be influenced by the few larger sized groups (despite the few small-sized groups) and the impact gets carried over to the overall test. In either the case of gender or race, there appears to be a greater amount of *within*-group inequality *between* groups even though the overall amount of inequality is greater between gender groups than between race groups in the BHPS data.

The 2005 Panel Study of Income Dynamics Example

For a second example, income data from the 2005 PSID family file are analyzed. The PSID is a US nationally representative longitudinal study of nearly 9,000 families that first began in

1968 and followed up every other year (though prior to 1987 the follow-up pattern was more complex), collecting data on economic, health, and social behavior. From these PSID data, three variables are included in the analysis of structural inequality: (previous years') income of the household head, gender, and race/ethnicity.

The income variable is a sum of the earnings from up to four jobs held by household heads in the previous year. The sex variable is used as is, and there were 5,139 male household heads (μ=$40,239) and 2,269 female household heads (μ=$16,074) from the 2005 PSID family file. There were also up to four mentions in terms of race, and there was a separate question about Spanish descent (which, in the US context, consists of Mexicans, Chicanos, Cubans, Puerto Ricans, and other Hispanics and Latinos. Combining the two race-related variables, a new ethnicity variable was constructed, resulting in a five-category grouping including 4,588 Caucasians (μ=$40,548); 165 Latinos ($\mu$=$24,084), 36 American Indians (μ=$24,026), 2,418 African Americans (μ=$18,996), 101 Asians and Pacific Islanders (μ=$49,665). In the construction of the ethnicity variable, African American affiliation gets priority assignment, with an identification with the race on any of the four mentions getting assigned to that category, as the way race-based stereotyping works in society. Using these social grouping variables, we analyzed the amount of structural household inequality and present the results in Table 3.

---Table 3 about here---

In either the case of gender or the case of race/ethnicity, there exists a significant amount of structural inequality, judging by the G_d and the $T_{s\cdot}$ measures. The G_d values are all sizable and are already greater than the G values when only nonzero values are used, let alone when zero and negative values are included. Over 1,500 cases have zero or negative values, hence making the impact of omitting these cases more consequential. Regardless of the range of values used, the $T_{s\cdot}$ for the various scenarios all give significant structural inequality estimates at least at the 0.001 level when the standard error estimates are applied. The conventional Theil decomposition downplays structural inequality somewhat: For example, Theil's L decomposition only allocates a bit less than 5% of the overall Theil to the between-group component whether gender or ethnicity is the grouping variable (though Theil's T decomposition gives a somewhere higher proportion). Even though T_{sb} gives a similar value range, their proportions out of the totals are much greater; in fact, both components of structural inequality are nonignorable because of their significance tests—the between-group inequality contrasted between groups and the within-group inequality contrasted between groups are neither ignorable, especially the component of (dispersion-based) within-group inequality compared between groups.

Finally, we consider both the sex and the race/ethnicity groupings together. The bottom two rows in Table 4 present the inequality results by analyzing the PSID 2005 data with 10 sex-by-race groups (i.e., Caucasian males, Caucasian females, …, Asian and Pacific Islander males, and Asian and Pacific Islander females). The overall G, T_T, and T_L of course remain the same as the race-based computations, regardless of the new groupings. The Theil between-group components remain a relatively small proportion of the total inequality even though they increased by a small amount. The G_d values are much greater than those from analyzing sex or race alone; all the structural Theil inequality measures are much more sizable than considering race or gender alone, suggesting that there exists a greater among of structural inequality when race intersects with gender. This is true for both the between-group inequality contrasted between groups and the within-group inequality contrasted between groups, indicating that the race *and* gender based structural inequality in the U.S., judging by the 2005 PSID data, are not important to considered, but they intersect each other. This conclusion cannot be drawn when only the conventional inequality measures are employed. Note that the amount of total structural inequality does not increase with the number of groups if there is no additional structural inequality—be it location-based or dispersion-based—*exposed* by adding or redefining the grouping variable. In the current case, having race-specific gender groups makes sociological sense and is supported by the additions in the amount of structural inequality captured.

The Example Comparing Six EU Countries Using Their 2000 Surveys

For a final example, we examine six EU countries, Austria, Belgium, France, Greece, Ireland, and Spain by using data from the Luxembourg Income Study, which harmonizes national survey data on income and wealth from many European countries and several countries from other parts of the world. For this example, we analyzed data on gender-based inequality in net income from these six countries. Net or post-tax income or wage data for sampled individuals were gathered on surveys from these countries, and positive income values were used in the analysis.

To get a sense of men's and women's income average differences, we summarize mean values for these countries below:

Nation: N_m; N_f	***Men's μ***	***women's μ***
Austria: 1,410; 943	264,300	179,400
Belgium: 1,296; 1,044	860,500	588,300
France: 5,820; 5,130	120,600	82,910
Greece: 1,477; 859	3,765,000	2,968,000
Ireland: 1,567; 1,227	16,420	11,350
Spain: 2,571; 1,530	2,552,000	1,948,000

Clearly, men's mean net income values were higher than women's in all six countries on their 2000 surveys. However, it would give us a better sense of inequality if we examine how these countries compare in terms of both conventional and structural inequality measures, which are presented in Table 4.

---Table 4 about here---

Judged by G, T_T, and T_L, the overall amount of inequality between individuals was highest in France and lowest in Belgium among these six countries according to the 2000 surveys. However, the aggregative between-group component of the overall inequality given by statistics such as G_a and T_{Tb} suggest that the inequality between men and women in these two countries are almost identical between each other and similar to the Austrian and Irish levels. In comparison, Greece had the lowest between-group aggregative measures even though its overall inequality was higher than two or three other countries (depending on the specific conventional inequality measure used). The structural inequality measures complement the picture with further information: Despite the consistency between G_a, T_{Tb} and T_{Lb} on the one hand and T_{sb} on the other, the ranking of the countries in total structural inequality in terms of T_{st} (to a lesser degree, G_d) is rather different from the aggregative between-group information, with France at the top, followed by a more or less three-way tie between Austria, Ireland, and Spain, then Greece, and with Belgium at the bottom. The difference in rank is largely due to the contributions of the between-group contrast of dispersion-related within-group inequality of these countries as France had the highest value in T_{sw} and Greece had the third highest. This six-country comparison shows that the structural inequality in terms of the differential dispersion-based within-group variation in income is ignored by conventional inequality measures regardless of how they are decomposed.

When conducting comparative research, researchers are often faced with the issue of dissimilar survey instruments and different social contexts. Whereas survey data can be harmonized, social contexts cannot. However, this should present no problem for analyzing structural inequality across countries. For example, even within the European Union, there exists much variation when it comes to racial/ethnic composition in society in both the number of racial/ethnic groups and the identity of these groups. Regardless of the number and the names of racial and ethnic groups, racial inequality represents a dimension in a society's structural inequality, and can be compared between countries as long as it is defined meaningfully and clearly for each country because the amount of structural inequality is not a function of the number (or the names) of the groups involved but is affected by the degree to which these social groupings are responsible for structural inequality.

The Socioeconomic Index Example

Our final example examines inequality in socioeconomic status between gender, race, and gender by race groups in the U.S., using the 2006 GSS data. The measurement of socioeconomic status employed by the GSS is based on procedures developed by Duncan (1961) and revised and updated by Nakao and Treas (1994) using the 1989 GSS study of occupational prestige. The concern in this soecioeconomic index (SEI) example is how inequality is affected by approximately similar mean levels of SEI but different spreads between the groups. We present below the levels of the group-specific mean and standard deviation SEI.

Group: *N*	***M***	***σ***
Men: 1,906	50.508	20.117
Women: 2,336	50.046	19.159
White: 3,135	51.373	19.658
Black: 575	44.134	18.025
White men: 1,426	51.701	20.258
White women: 1,709	42.673	17.257

Black men: 217	51.099	19.144
Black women: 358	45.020	18.442

From these descriptive statistics, one can see that there was little gender difference in mean SEI though men and women differed in within-group spread of the SEI in 2006. The two racial groups differed in both the mean and the standard deviation of the SEI, and heterogeneity was present among the cross-classified race-gender groups.

In Table 5, we present the comparison between the conventional individual inequality measures and the proposed structural inequality measures. The classic Gini (G) index and the between-group Gini (G_a) reveal a moderate amount of inequality in SEI for all groupings. The structral Gini (G_d) shows a sizable difference for the cross-classified race-gender groups though not for the gender- or race-based analysis.

---Table 5 about here---

All conventional Theil measures indicate a small amount of inequality. However, since all these amounts are relative, we must take a closer look at how statistically significant the results are. The between-gender socioeconomic inequality is almost entirely driven by the structural-within component (T_{sw}), due primarily to the differing spread in the group-specific SEIs. The between-race socioeconomic inequality, on the other hand, mainly hinges on the structural-between component (T_{sb}), due primarily to the differing average in the group-specific SEIs. When race and gender groups are cross-classified, both forms of socioeconomic inequality become salient—with both the structural-within and the structural-between components contributing significantly to the total structural socioeconomic inequality in the U.S.

The GSS SEI example suggests that the structural Theil measures provide an added benefit over the structural Gini measure, which, though having a higher value, does not lead to any conclusive evidence of a different kind of structural differentiation. Using the structural Theil statistics, we can conclude that the between-gender socioeconomic inequality in the U.S. in 2006 was primarily driven by heteroskedasticity, or differential dispersions that are group-specific; the between-race socioeconomic inequality was determined mainly by between-group aggregate mean differences; the between-race-and-gender socioeconomic inequality was given rise by both group mean differences and by heteroskedasticity.

Conclusion

In this paper we conceptualized inequality in a population as having two basic types—individual inequality and structural inequality—with the former generated by individual variations and the latter responding to the social structure in a society represented by social groups. The concept was further defined as a function of income share, population share and weight. We proposed a family of structural inequality measures, with a structural Gini version and a structural decomposable Theil version and its variance estimator as particular implementations.

The proposed measures were examined in five applications. The first was a simulation study, where a range of two-class situations of varying degrees of inequality were analyzed. The simulation study showed the G_d measure complements the information obtained form G and G_d and that the structural Theil measures $T_{s\cdot}$, together with their standard errors, can accurately assess the amount of structural inequality with good sensitivity.

The four empirical data analyses using the 1991 British Household Panel Survey, the 2005 U.S. Panel Study of Income Dynamics, the 2000 Luxembourg Income Study, and the 2006 U.S. General Social Survey data demonstrated further useful properties of the proposed measures. First of all, the second empirical example suggests that multiple social groupings may give rise to a different amount of structural inequality even though the amount of individual inequality is not affected. This is important for applied research because invariably in almost all countries the existence of multiple social groupings gives rise to structural inequality—think about any of the nations in the world, where there is at work always a possible combination of stratification factors of gender, ethnicity, nativity, language, religion, and so on.

In addition, the third empirical example demonstrated that the measures are useful for comparative research in particular: The rank-order of total structural inequality, when compared among countries, is different from the rank-order obtained from the conventional inequality measures (due to the exclusion of an important type of inequality—that of between-group comparison of within-group differentiations).

The final empirical example of the GSS data measuring SEI further illustrates the usefulness of the proposed structural inequality measures. In this case, the groups may appear to have identical mean values in SEI while their dispersions can be different. This property, neglected by the conventional measures, is captured by the new measures, in particular the structural Theil measures, assisted by their standard errors for assessing statistical significance.

In summary, the concept of structural inequality is operationalized and implemented in the structural Gini and the structural Theil measures in this paper. The same principle and operation should be able to extend to other inequality measures such as VarLog (Firebaugh 1999) and the Atkinson index, among others. Each implementation may give different insight from each other and from the conventional measures.

The applications in the paper taken together suggest some useful properties of the new measures that the conventional Gini and Theil measures including Theil decompositions do not possess. The proposed measures are consistent with Blau's theory on inequality and heterogeneity, which focuses on the intersection between horizontal and vertical differentiations. That is, vertical differentiations in rewards described by horizontal distributions define structural inequality that has two components—a location-based component and a dispersion-based component. In other words, structural inequality measures reflect a component of group-specific locations compared between groups (location-between) and a component of group-specific dispersions contrasted between groups (dispersion-between). Even though the idea of a location-between measure is similar to the conventional between-component of the Theil, the dispersion-between and thus the total-between measures have no precedent. Used together with the conventional measures, the structural inequality measures provide a complementary picture of the structure of inequality in a society; standing alone, these measures provide a single index of the degree of stratification in the form of how vertical distributions is influenced by horizontal distributions that is ignored by conventional measures. When using the structural inequality measures alone, we may also include multiple groupings—some key factors from a list of group identities such as gender, ethnicity, nativity, language, and religion—simultaneously to obtain a more complete sense of structural inequality. This suggests that, commonly reported Gini ratios, as interpretable as they may be, may be misleading when it comes to how inequality is structurally generated. The knowledge from structural inequality measures can help us understand and assess inequality in its true form.

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Table 1: Comparing Individual and Structural Inequality Measures for Five Distributions Described by the Pareto Principle (standard errors in italics)

Data	G	G_d	G_a	T_T	T_{Tb}	T_{Tw}	T_L	T_{Lb}	T_{Lw}	T_{st}	T_{sb}	T_{sw}
D_1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
										0.126491	*0.005657*	*0.178885*
D_2	0.480000	0.780234	0.480000	0.502065	0.502065	0.000000	0.502065	0.502065	0.000000	0.502065	0.502065	0.000000
										0.129988	*0.005813*	*0.183831*
D_3	0.648000	0.912738	0.648000	1.000309	1.000309	0.000000	1.000309	1.000309	0.000000	1.000309	1.000309	0.000000
										0.139374	*0.006233*	*0.197104*
D_4	0.834000	0.983748	0.834000	2.003491	2.003491	0.000000	2.003491	2.003491	0.000000	2.003491	2.003491	0.000000
										0.176056	*0.007873*	*0.248980*
D_5	0.968000	0.999471	0.968000	3.987228	3.987228	0.000000	3.987228	3.987228	0.000000	3.987228	3.987228	0.000000
										0.362164	*0.016196*	*0.512177*
D_6	0.998000	0.999998	0.998000	6.892941	6.892941	0.000000	6.892941	6.892941	0.000000	6.892941	6.892941	6.892941
										1.416337	*0.063341*	*2.003003*

Table 2: Comparing Individual and Structural Inequality Measures, the 1991 BHPS Data

Grouping	G	G_d	G_a	T_T	T_{Tb}	T_{Tw}	T_L	T_{Lb}	T_{Lw}	T_{st}	T_{sb}	T_{sw}
Sex, *N*=5,288	0.404793	0.480977	0.129112	0.272293	0.044274	0.228018	0.308415	0.050490	0.257925	**0.144776**	**0.047382**	**0.097394**
adj. HH	0.360346	0.392113	0.083495	0.216194	0.017705	0.198489	0.227127	0.019016	0.208111	**0.099876**	**0.018360**	**0.081516**
adj. HH/h	0.357177	0.381814	0.073306	0.213209	0.013527	0.199682	0.221833	0.014370	0.207463	**0.096038**	**0.013949**	**0.082090**
Race, *N*=5,085	0.405466	0.428991	0.005914	0.273381	0.000700	0.272681	0.308224	0.000695	0.307529	**0.016878**	**0.000697**	0.016180
adj. HH	0.359598	0.395472	0.005032	0.215326	0.001028	0.214298	0.225187	0.001268	0.223919	**0.012487**	**0.001148**	0.011339
adj. HH/h	0.356406	0.393252	0.004864	0.212337	0.001030	0.211306	0.219876	0.001283	0.218593	**0.012263**	**0.001157**	0.011106

Note: Abbreviations are used are adj. HH=income needs-adjusted with household size, and adj. HH/h =income adjusted by household size and housing need; boldface indicates estimates are at least twice their standard errors.

Table 3: Comparing Individual and Structural Inequality Measures, the 2005 PSID Data

Grouping	G	G_{d}	G_{a}	T_T	$T_{T\mathrm{b}}$	$T_{T\mathrm{w}}$	T_L	$T_{L\mathrm{b}}$	$T_{L\mathrm{w}}$	T_{st}	T_{sb}	T_{sw}
Sex, *N*=5,889	0.511220	0.551958	0.120435	0.548129	0.041860	0.506269	1.021447	0.048540	0.972907	**0.317767**	**0.045200**	**0.272567**
***N*=7,408**	0.643496	0.698682	0.156363	-	0.065688	-	-	0.077800	-	-	**0.071744**	-
Race, *N*=5,814	0.511908	0.673218	0.145201	0.550535	0.048498	0.502038	1.021890	0.054000	0.967890	**0.409742**	**0.051249**	**0.358493**
***N*=7,308**	0.631278	0.830866	0.153609	-	0.053008	-	-	0.059169	-		**0.056088**	-
SexRace, *N*=5,814	0.511908	0.741104	0.211393	0.550535	0.074431	0.476104	1.021890	0.081822	0.940068	**0.529477**	**0.078127**	**0.451351**
***N*=7,308**	0.631278	0.955503	0.254940	-	0.101197	-	-	0.112470	-		**0.106833**	-

Note: Boldface indicates estimates are at least twice their standard errors.

Table 4: Comparing Sex-Based Structural Inequality in Net Wage among Six 2000 LIS Countries

Grouping	G	G_d	G_a	T_T	T_{Tb}	T_{Tw}	T_L	T_{Lb}	T_{Lw}	T_{st}	T_{sb}	T_{sw}
Austria, *N*=2,353	0.286534	0.309686	0.088548	0.146933	0.016836	0.130097	0.161029	0.017475	0.143554	**0.080572**	**0.017156**	**0.063416**
Belgium, *N*=2,340	0.244263	0.266786	0.091012	0.107722	0.017087	0.090635	0.112395	0.017524	0.094870	**0.062067**	**0.017306**	**0.044761**
France, *N*=10,950	0.384088	0.396327	0.091210	0.263021	0.016932	0.246089	0.359177	0.017263	0.341914	**0.162466**	**0.017097**	**0.145369**
Greece, *N*=2,336	0.296764	0.298131	0.053362	0.160534	0.006265	0.154269	0.180712	0.006425	0.174287	**0.076597**	**0.006345**	**0.070253**
Ireland, *N*=2,794	0.303051	0.321066	0.087876	0.153186	0.015996	0.137191	0.167936	0.016415	0.151521	**0.085964**	**0.016206**	**0.069758**
Spain, *N*=4,101	0.310564	0.323288	0.060663	0.169711	0.008068	0.161643	0.177899	0.008300	0.169599	**0.084472**	**0.008184**	**0.076288**

Note: Boldface indicates estimates are at least twice their standard errors.

Table 5: Comparing Individual and Structural Inequality Measures, the 2006 GSS Socioeconomic Index Data

Grouping	G	G_{d}	G_{a}	T_T	$T_{T\mathrm{b}}$	$T_{T\mathrm{w}}$	T_L	$T_{L\mathrm{b}}$	$T_{L\mathrm{w}}$	T_{st}	T_{sb}	T_{sw}
Sex, *N*=4,242	0.223295	0.223841	0.001192	0.076983	2.87e-06	0.076980	0.078829	2.87e-06	0.078826	**0.038581**	2.87e-06	**0.038578**
Race, *N*=3,710	0.220093	0.229032	0.018865	0.074720	0.001407	0.073313	0.076818	0.001458	0.075360	0.020193	**0.001433**	0.018761
SexRace, *N*=3,710	0.220093	0.339165	0.025767	0.074720	0.001467	0.073253	0.076818	0.001525	0.075293	**0.047775**	**0.001496**	**0.046279**

Note: Boldface indicates estimates are at least twice their standard errors.

Figure 1: Histograms of the 15 Sets of Simulated Data

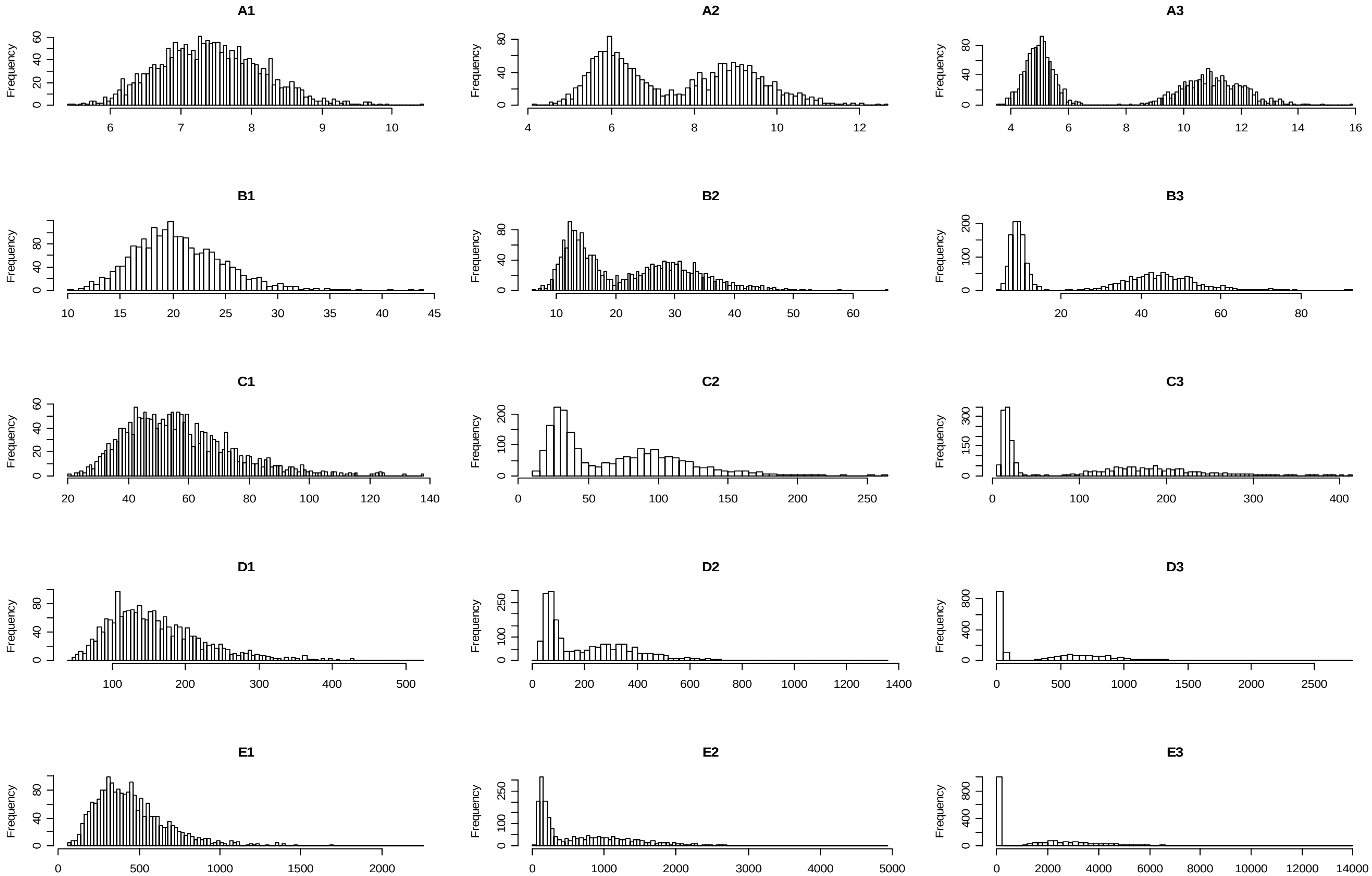
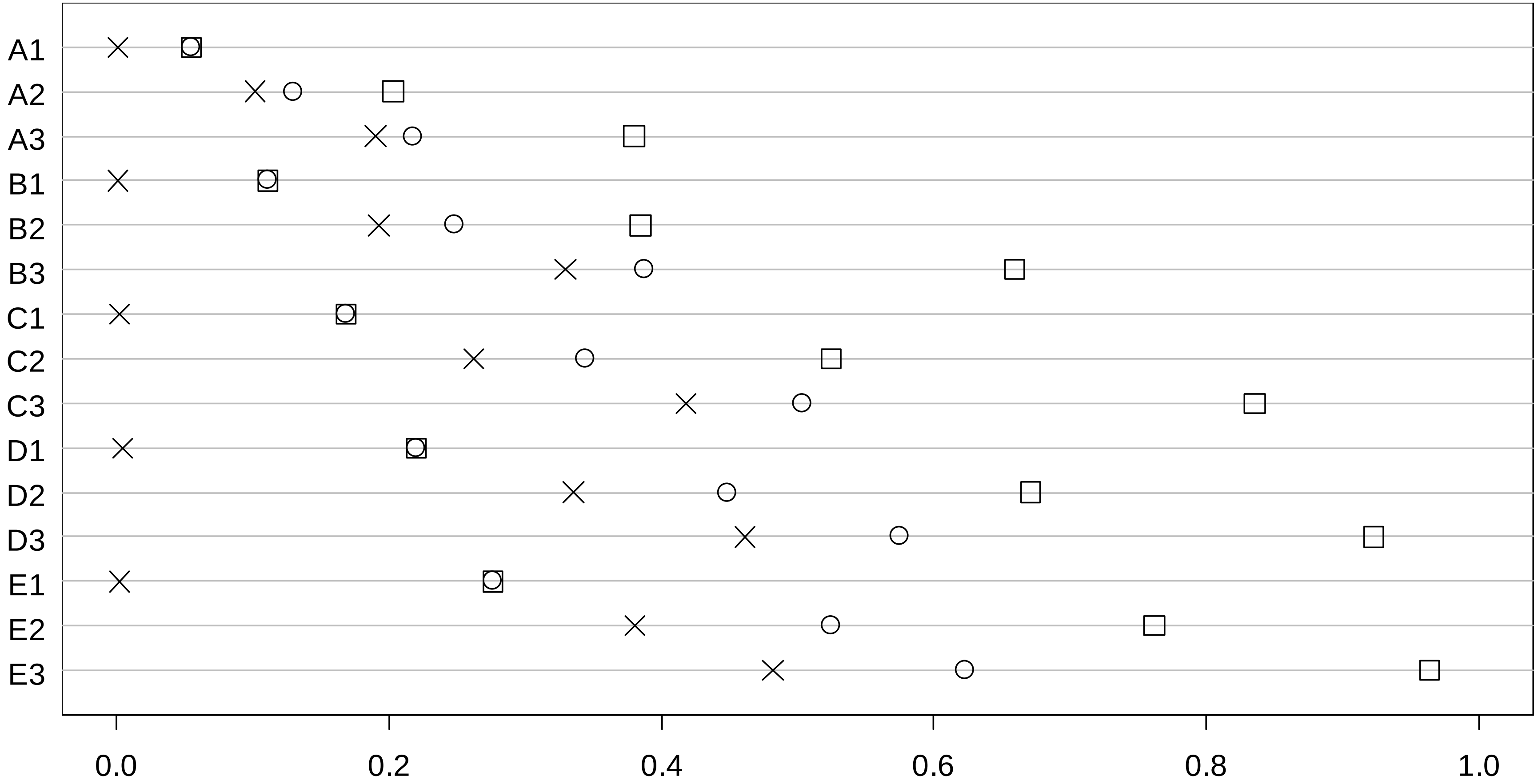
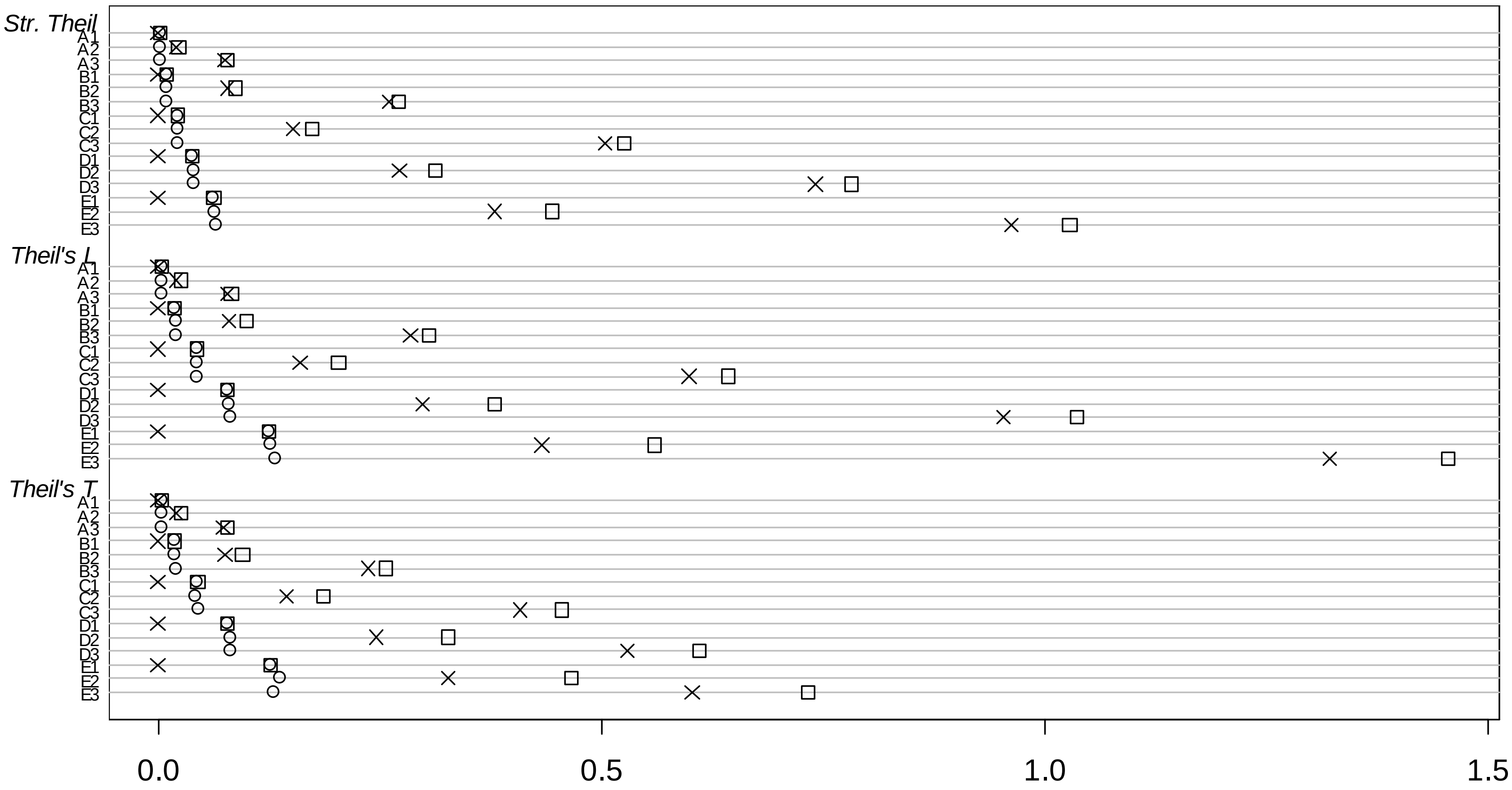


Figure 2: Comparing Individual and Structural Gini Measures of the 15 Simulated Distributions



Note: ○ = Gini index; × = aggregate between-group Gini; □ = structural Gini.

Figure 3: Comparing Individual and Structural Theil Measures of the 15 Simulated Distributions



Notes: □ = Theil total; × = Theil between component; ○ = Theil within component. These structural Theil statistics are at least twice of their standard errors: □ for B_2, B_3, C_2, C_3, D_1, D_2, D_3, E_1, E_2, and E_3; × for A_2, A_3, B_2, B_3, C_2, C_3, D_2, D_3, E_2, and E_3; ○ for D_1, D_2, D_3, E_1, E_2, and E_3.